

Security Design in a Production Economy with Flexible Information Acquisition*

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This Version: September, 2013

First Draft: June, 2012

Abstract

This paper looks into security design in a production economy with a new informational friction. It highlights the investor as an expert, who may acquire costly information on the market prospect of the entrepreneur's project and then screen it through financing decisions. Thus, real production depends on information production, while these two are separated, which constructs a friction. Debt is optimal when the dependence of real production on information production is weak and thus the friction is not severe, and convertible preferred stock is optimal when the dependence is strong and thus the friction is severe. Such a dichotomy is unified under a new interaction between the payoff structures of securities and the flexible incentives to screen regarding attention allocation. Both the optimal securities and their correspondences to economic environments fit in line with empirical evidence. A new approach, flexible information acquisition, characterizes information sensitiveness of securities and attention allocation of investors in a state contingent way, which enables us to work with arbitrary securities on continuous states without distributional assumptions and delivers sharper predictions.

KEYWORDS: security design, production economy, flexible information acquisition.

JEL: D82, D86, G24, G32, L26

*Earlier versions have been circulated under the title, "Venture Finance under Flexible Information Acquisition." We thank Malcolm Baker, John Campbell, Peter DeMarzo, Emmanuel Farhi, Paolo Fulghieri, Diego Garcia (discussant), Simon Gervais, Barney Hartman-Glaser, Ben Hebert, Steven Kaplan, Arvind Krishnamurthy, Josh Lerner, Stephen Morris, Marcus Opp, Jonathan Parker, Raghuraj Rajan, Adriano Rampini, David Robinson, Hyun Shin, Andrei Shleifer, Alp Simsek, S. Viswanathan, and Michael Woodford; and seminar and conference participants at Berkeley Haas, Duke Fuqua, Harvard, MIT, UNC Kenan-Flagler, Peking University, Stanford GSB, 2013 Finance Theory Group Workshop, 2013 WFA Annual Meeting, and 2013 Toulouse TIGER Forum for helpful comments.

1 Introduction

The typical landscape of corporate finance is that an entrepreneur comes up with a business plan, and then gets finance, under specific contract terms, from an investor to start a project. The entrepreneur is often viewed as an expert who is more informed. Nevertheless, an important aspect missed in this prevailing approach is that investors are often more capable of assessing projects' uncertain market prospects by their industry experience, and thus help screen projects through financing decisions. For instance, a medical professor who invents a drug seeks venture finance for large scale production, and the venture capitalist acquires information to evaluate its market prospect and decides whether to finance it. A chef who considers opening a restaurant goes to a local bank for a loan, and the bank may acquire market information about its popularity based on past experiences in financing various local restaurants. In these examples, the entrepreneurs initiate different forms of contracting by inviting investors and designing securities. But the investors may be more informed through acquiring information and screening the project, which in turn affects the entrepreneurs' security design. As suggested by [Tirole \(2006\)](#), a disadvantage of conventional corporate finance literature is that such information advantage by investors is neglected.¹ Our paper fills the blank by providing a unified framework to understand why and how various projects are financed by issuing various securities, through uncovering the role of investors' information acquisition in the financing process.²

Highlighting investors' ability to acquire information, this paper studies security design in a production economy with friction. An entrepreneur can exclusively undertake a risky project, but she has no money for the investment. However, she may try to get finance through contracting with an investor. Importantly, the investor can acquire costly information about the project's uncertain future cash flow before making a financing decision. Only when the investor believes the project is good enough, can the project be financed. In other words, the investor screens the project through information acquisition. Therefore, real production depends on information production, but they are separately performed by the two agents, which constructs the fundamental friction in this production economy. Due to this friction, the entrepreneur confronts two conflicting forces. On the one hand, it is necessary for the entrepreneur to reward the investor, because it induces the investor to screen the project more effectively, leading to a higher social surplus. On the other hand, as the entrepreneur shares the social surplus with the investor, she also wants to retain as much as possible. The optimal security reduces the friction by reconciling the two forces.

¹Some exceptions are surveyed by [Bond, Edmans and Goldstein \(2012\)](#). However, none of these papers speaks to the context of security design. A burgeoning security design literature highlights investors' information advantage ([Axelson, 2007](#); [Dang, Gorton and Holmstrom, 2011](#); [Yang, 2012a](#); [Hennessy, 2012](#)), but these papers are pertaining to the asset-backed securities market and not fit for the corporate finance setting with production.

²Throughout the paper, the terms information acquisition, screening, and information production share the same technical specification. Henceforth, we use them interchangeably to ease understanding.

We formulate and unify the optimal securities that are used to finance various projects differing in nature. They correspond to standard debt and convertible preferred stock, which further construct new pecking orders along different dimensions.³ In solving for these optimal securities, we do not have restrictions on the spaces of feasible securities, priors, or information structures. The fundamental is built over continuous states with arbitrary distributions, as opposed to finite discrete states or continuous states with given distributional assumptions often seen in previous literature. Hence, the predictions are sharp and robust. Our predictions also help bridge the security design literature and the classic pecking order theory (Myers and Majluf, 1984), because our pecking order of optimal securities comes from an optimization over a general security space, as opposed to an exogenously given set of securities like debt and equity.

When the dependence of real production on information production is weak, namely, the friction is not severe, the optimal security is debt that does not induce the investor to acquire information. This case corresponds to the scenarios when the project's ex-ante market prospect is already good or the cost of screening is high. Thus, the benefit of screening by information acquisition does not justify the cost. Therefore, it is optimal to deter costly information acquisition by issuing debt, the least information sensitive security. Interestingly, our intuition to render debt optimal is different from the conventional wisdom of information insensitiveness, as our mechanism does not feature adverse selection.⁴ This is consistent with the evidence that many conventional businesses and less revolutionary start-ups rely heavily on traditional debt finance (for example, Petersen and Rajan, 1994; Kerr and Nanda, 2009).

In contrast, when the dependence of real production on information production is strong, namely, the friction is severe, the optimal security turns out to be participating convertible preferred stock that induces the investor to acquire information. This prediction is new to the security design literature, but is closely in line with empirical facts. As documented in Kaplan and Stromberg (2003), four fifths of the contracts between entrepreneurs and venture capitals are convertible preferred stocks, and half of them are participating convertible preferred stocks. The predicted multiple of the convertible preferred stock, defined as the ratio of its face value to the investor's initial investment, is always larger than one, which also matches the empirical documents in Lerner, Leamon and Hardymon (2012). This case corresponds to the scenarios when the project's ex-ante market prospect is not good or the cost of screening is low. Information is valuable as it updates the investor's perceived market prospect of the project, and thus may help screen in a potentially good project or screen out a potentially bad project. An information-

³Our notion of pecking order means the order of optimal securities over the dimension of a certain parameter. It is more general than the classic concept in Myers and Majluf (1984), featuring the dimension of financing cost.

⁴Notable results regarding debt as the least information sensitive security to mitigate adverse selection include Myers and Majluf (1984); Gorton and Pennacchi (1990); DeMarzo and Duffie (1999); Dang, Gorton and Holmstrom (2011); Yang (2012a). None of those works considers production.

adjusted expected net present value (NPV) criterion is offered, as a project with ex-ante negative expected NPV may also be financed. This is consistent with the idea that investors use their specific industry experience to screen projects. As a result, the entrepreneur is willing to compensate the investor to encourage such beneficial screening. Specifically, the entrepreneur designs a security that pays generously in general, but differently across states, which encourages the investor to acquire adequate information to distinguish between states. This also suggests that standard equity is not optimal because it pays too little in bad states, which fits the reality that standard equity is the least used security in financing new projects (Lerner, Leamon and Hardyman, 2012).

Interestingly, the investor may acquire information no matter whether the project's ex-ante NPV is positive or negative, but the implications of screening are different. A positive NPV project would be financed for sure if the investor could not screen it. Facing an investor who can screen, however, the entrepreneur may propose convertible preferred stock that leads to screening. A potentially bad project may be screened out, and thus the financing cost may be reduced. In contrast, a negative NPV project would be rejected for sure if the investor could not screen it. With information acquisition, a potentially good project may be screened in. Either screening out or in, the entrepreneur gets better off than the counterfactual where the investor only provides dumb money, and convertible preferred stock is used to encourage favorable information acquisition.

The friction-based pecking order of debt and convertible preferred stock may also be mapped to three empirical dimensions to deliver more intuitions: the profitability of the project, the uncertainty of the project, and the cost of screening. We perform comparative statics of optimal securities along the three dimensions. These comparative statics further suggest that different projects are endogenously financed by different securities, and potentially different types of investors. The role of screening varies as well, but is still unified under the friction: the different extent of dependence of real production on information production.

Our parsimonious framework can accommodate a variety of theoretical corporate finance contexts as well as real-world scenarios of financing entrepreneurial production. On the one hand, we highlight the investor as an expert on screening. The acknowledgement of investors' information expertise dates back to Knight (1921) and Schumpeter (1942). Besides numerous modern anecdotal evidence (see Kaplan and Lerner, 2010; Da Rin, Hellmann and Puri, 2011, for reviews), recent empirical literature (Kerr, Lerner and Schoar, 2011; Chemmanur, Krishnan and Nandy, 2012) have also acknowledged the presence of screening by various institutional investors. As the cost of screening in our model pertains to both the project's nature and the investor's information expertise, it allows us to cover various investors, including venture capitals, angel investors, specialized banks, and other financial institutions. On the other hand, we highlight two aspects of the entrepreneur. First, the entrepreneur has access to the technology but is financially con-

strained. Second, the entrepreneur’s human capital is inalienable, which means the investor can neither buy the technology nor hire the entrepreneur as a worker, and the entrepreneur initiates contracting by designing and proposing the security.⁵ These settings fit with the argument in [Rajan \(2012\)](#) that the differentiation of entrepreneurs is important in the early stage of firms’ life cycles. This paper, to the best of our knowledge, is the first to investigate the interplay between information acquisition and security design in a production economy, and deliver predictions that are both sharp and consistent with empirical evidence regarding the contracts between real-world entrepreneurs and investors ([Sahlman, 1990](#); [Gompers, 1999](#); [Kaplan and Stromberg, 2003](#)).

A comparison between the production economy and an exchange economy helps highlight the essential relationship between security design and information acquisition. [Yang \(2012a\)](#) considers information acquisition in an exchange economy and focuses on the unique optimality of debt in the asset-backed securities (ABS) market. In that model, a seller has an asset in-place and proposes an asset-backed security to a buyer to raise liquidity. The buyer can acquire information about the fundamental before her purchase. Debt is shown to be uniquely optimal in this case because it is least information sensitive and mitigates the buyer’s adverse selection to the greatest extent. In this exchange economy with an asset in-place, the social surplus depends negatively on information acquisition. This is because that, first, information acquisition leads to endogenous adverse selection, and second, information is costly per se. As a result, to discourage information acquisition is desirable. On the contrary, in this paper, the entrepreneur’s project can only be initiated if the investor provides finance. Thus, our model features a production economy in which the social surplus may depend positively on information acquisition. In this case, adverse selection is no longer the focus. Instead, the entrepreneur wants to design a security that encourages the investor to acquire information in favor of herself. Therefore, debt may no longer be optimal when information acquisition is desirable.

A new concept, flexible information acquisition, plays an important role in establishing our sharp predictions on the payoff structure, visually, the “shape,” of optimal securities, which cannot be achieved in previous literature.⁶ Facing securities with different shapes, the investor is incentivized to allocate costly attention in different states of the underlying cash flow while screening. For instance, a debt holder allocates her attention to bad states, as her payoff is constant over good states so that only the default risk matters, while an equity holder pays more attention to good states as she benefits from the upside payments. For an arbitrary security, the investor’s incentive of attention allocation in screening would be determined in this state contingent way

⁵We have also relaxed this assumption to ensure the robustness of our results. The results of optimal securities are unaffected, either if the project is transferrable or if the entrepreneur does not have full bargaining power in designing the security.

⁶Also see [Yang \(2012a\)](#) for a justification of applying flexible information acquisition on security design.

accordingly, and in turn affects the entrepreneur’s incentive to design the security. In characterizing these incentives, the traditional approach of exogenous information asymmetry or exogenous inattention is inadequate. Moreover, recent models of endogenous information acquisition also fail to accommodate such flexibility of incentive, since they only consider the amount or precision of information (see [Veldkamp, 2011](#), for a review).⁷ In contrast, our new approach following [Yang \(2012a,b\)](#) allows agents to choose not only how much, but also what kind of information to acquire. It employs rational inattention ([Sims, 2003](#); [Woodford, 2008](#)) as a foundation, but with different focuses. The original rational inattention concept mainly captures the bounded rationality of agents: they have limited capacity of attention and cannot pay full attention to all payoff-related variables. Instead, flexible information acquisition highlights the opposite: agents have the capacity to allocate attention to different states of the economy in a flexible way. This renders flexible information acquisition a match to strategic interactions like security design, while the original rational inattention works for decision problems to generate rigidity-related phenomena.

Related Literature. This paper is closely related to a series of theoretical work that predicts non-debt-like optimal securities in various circumstances with information asymmetry (see [Brennan and Kraus, 1987](#); [Boot and Thakor, 1993](#); [Nachman and Noe, 1994](#); [Fulghieri and Lukin, 2001](#); [Inderst and Mueller, 2006](#); [Axelson, 2007](#); [Chakraborty and Yilmaz, 2011](#); [Dow, Goldstein and Guembel, 2011](#); [Hennessy, 2012](#); [Fulghieri, Garcia and Hackbarth, 2013](#)). Compared to those papers, all of which focus on ex-ante information asymmetry or rigid information acquisition, our model delivers clearer interaction between security design and the endogenous and flexible screening. Also, the models in previous literature can only accommodate discrete states, or continuous states with distributional assumptions,⁸ or restricted sets of feasible securities.⁹ Thanks to flexible information acquisition, we can model arbitrary securities on continuous states with arbitrary distributions and thus to characterize the shapes of optimal securities more specifically. Through this line, our model solidifies the classic intuition of information sensitiveness that non-debt-like securities may encourage investors to acquire information.

A new strand of literature on the real effects of rating agencies in a production economy context (see [Kashyap and Kovrijnykh, 2013](#); [Opp, Opp and Harris, 2013](#)) is also related. On behalf of investors, the rating agency screens the firm, who does not know its type. Information acquisition may improve social surplus through ratings and the resulting investment decisions. Different from this literature, we study how different shapes of securities interact with the incentives to allocate attention in acquiring information and as a result the equilibrium financing choice. Flexible

⁷Technically, we call those information acquisition technologies rigid, because they impose parametric restrictions on the signals, while flexible information acquisition allows for any conditional distribution over the fundamental.

⁸For example, the monotone likelihood ratio property (MLRP).

⁹Most of the security design literature only consider monotone securities, or even monotone residuals.

information acquisition also allows us to combine the two roles of rating agencies and investors, and thus to flesh out the impact of endogenous screening on security design.

Our model contributes to the literature of venture contract design by highlighting information acquisition, which is overlooked in past research. Although security design is the focus of modern research in innovation and entrepreneurial finance, existing literature mainly pay attention to monitoring and moral hazard ([Schmidt, 2003](#); [Casamatta, 2003](#); [Hellmann, 2006](#)), refinancing and staging of finance ([Bergemann and Hege, 1998](#); [Cornelli and Yosha, 2003](#); [Repullo and Suarez, 2004](#)), as well as allocation of control rights ([Berglof, 1994](#); [Hellmann, 1998](#); [Kirilenko, 2001](#)), leaving the role of screening untouched. Also, existing models often fail to deliver consistent theoretical predictions with real-world securities.¹⁰

Our model also delivers implications on the impact of information on liquidity in various markets. In our production economy, the aggregate cash flow depends on the financing decision, so that the circumstance also features a primary financial market. In contrast, the circumstance of [Yang \(2012a\)](#), where the aggregate cash flow is fixed, features an exchange economy or a secondary financial market. Earlier mechanism design literature pertaining to liquidity and information gathering also highlight such comparison between different economies or financial markets, and suggest that the contribution of information on liquidity would differ accordingly ([Cremer and Khalil, 1992](#); [Cremer, Khalil and Rochet, 1998a,b](#)). But these papers do not focus on security design and cannot predict the forms of optimal contracts in different markets.

Finally, a growing literature focusing on the strategic interaction between firms and consumers also highlights endogenous attention allocation. Close to our setting is [Bordalo, Gennaioli and Shleifer \(2013\)](#) arguing that consumers may pay different attention to different product attributes, and such allocation is determined by product design and competition.¹¹ Complementing this literature, flexible information acquisition focuses on costly attention allocation across states, and delivers insights within the standard Bayesian updating framework commonly used in finance.

The rest of the paper is organized as follows. Section 2 specifies the environment of the production economy and elaborates the concept of flexible information acquisition. The optimal securities are characterized and discussed in Section 3. Section 4 gives both the theoretical and numerical results on our new pecking orders. Section 5 performs comparative statics on the optimal securities. In the final section, we conclude and discuss possible directions for further research. Unless otherwise noted, all proofs are attached in the Appendix.

¹⁰For instance, as documented in [Da Rin, Hellmann and Puri \(2011\)](#), the most commonly used double moral hazard models are able to approximately predict the contracts between entrepreneurs and venture capitals, but these models cannot deliver the precise allocation of cash flow rights between the two parties.

¹¹An implication of this mechanism in financial markets is that financial innovations may draw investors' attention to returns instead of risks, which leads to neglected risks highlighted by [Gennaioli, Shleifer and Vishny \(2012, 2013\)](#).

2 Model

We present our stylized model of a production economy, focusing on the interplay between security design and flexible screening. We make assumptions to highlight the key friction: the dependence and separation of real production and information production.

2.1 Financing Entrepreneurial Production

Consider an economy with two dates, $t = 0, 1$, and a single consumption good. There are two agents: an entrepreneur and a deep-pocket investor, both risk neutral. Their utility function is the sum of consumptions over the two dates:

$$u = c_0 + c_1,$$

where c_t denotes an agent's consumption at date t . We assume that the entrepreneur starts with zero wealth, while the deep-pocket investor has large endowment at date 0. In what follows we use subscripts E and I to indicate the entrepreneur and the investor, respectively.

We consider the finance of the entrepreneur's risky project. To initiate the project at date 0, the underlying technology requires an investment $k > 0$. If financed, the project generates a non-negative verifiable random cash flow θ at date 1. The project cannot be initiated partially. Hence, the entrepreneur has to raise k , by selling a security to the investor at date 0. The payment of a security at date 1 is a mapping $s : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $s(\theta) \in [0, \theta]$ for any θ . We only focus on the cash flow aspect of projects and securities.

We specify the processes of security design and information acquisition, both at date 0. The agents have a common prior Π on the future cash flow θ , if the project is financed. The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to the investor at price k . Facing the offer, the investor acquires information about the future cash flow θ in the manner of rational inattention (Sims, 2003; Woodford, 2008; Yang, 2012a,b), updates her belief on θ , and then decides whether to accept the offer. This process is modeled by flexible information acquisition, where the information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is μ , which is interpreted as the cost of screening. We will elaborate flexible information acquisition in the next subsection.

The implicit assumptions in the settings are reasonable to capture the key features in financing entrepreneurial production, especially from a perspective of screening. First, the entrepreneur can only undertake the risky project if she gets financing. This is consistent with earlier evidence that entrepreneurs are often financially constrained (Evans and Jovanovic, 1989; Holtz-Eakin, Joulfaian and Rosen, 1994). Even in mature firms, managers seek for outside finance since the

internal capital market often does not function well for new risky projects (Stein, 1997; Scharfstein and Stein, 2000). Second, more importantly, the investor is able to acquire information about the cash flow and thus screen the project through her financing decision. This point not only accounts for the empirical evidence, but also stands our framework out from most previous security design literature that feature the entrepreneur’s exogenous information advantage. Taking these two points together leads to the dependence and separation of real production and information production, which is the key friction in our framework.

It is worth noting what aspects of the finance in a production economy are abstracted away, and to what extent they affect our work. First, to focus on screening, we put moral hazard aside. To ignore moral hazard is a general practice in the security design literature, especially when information is highlighted (see DeMarzo and Duffie, 1999; DeMarzo, 2005, for a justification). Second, we assume that the entrepreneur’s human capital is inalienable, so that she has the bargaining power to design a security and a project transfer is impossible. Together with the differentiation argument in Rajan (2012), this assumption also broadly corresponds to the earlier incomplete contract literature which suggest the ownership go to the entrepreneur when innovative firms are young (Aghion and Tirole, 1994). In a later section we will formally show that even if the project is transferrable, to transfer the project at any fixed price is always not optimal. Moreover, in the Appendix we discuss a general allocation of bargaining powers between the two agents and we show that it does not impact our main results. Third, the bargaining is abstracted as a take-it-or-leave-it offer from its reality. This implies that the allocation of control rights is not our focus. Fourth, we do not model the staging of finance, and thus we interpret the cash flow θ in our model as already taking the consequence of investors’ exiting into account. Last, investors are unlikely to be risk neutral in reality. Nevertheless, risk neutrality enables us to first focus on screening as opposed to risk sharing, which is of less interest for our purpose. Our results would be strengthened if we take investors’ risk aversion into account.

2.2 Flexible Information Acquisition

We model the way through which the investor screens the project by flexible information acquisition (Yang, 2012a).¹² It allows us to work with arbitrary securities over continuous states without distributional assumptions and deliver sharp predictions, which stands our paper out of most existing security design literature. Fundamentally, the entrepreneur is able to design the security’s payoff structure in an arbitrary way, which may result in arbitrary allocation of attention for the investor when she screens the project. It thus calls for an equally flexible account of screening to capture the interaction between the shape of securities and the incentive to allocate attention.

¹²For more detailed expositions of flexible information acquisition, see Woodford (2008) and Yang (2012a,b).

This goal is not achievable through classic information acquisition technologies.

The key of flexible information acquisition is that it captures not only how much but also what aspects of information that an agent acquires. Consider an agent who chooses a binary action $a \in \{0, 1\}$ and receives a payoff $u(a, \theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure Π over \mathbb{R}_+ . Before making a decision, the agent may acquire information through a set of binary-signal information structures, each signal corresponding to one optimal action.¹³ Specifically, she may choose a measurable function $m : \mathbb{R}_+ \rightarrow [0, 1]$, the probability of observing signal 1 if the true state is θ , and acquire binary signals $x \in \{0, 1\}$ parameterized by $m(\theta)$. $m(\theta)$ is chosen to ensure that the agent's optimal action is 1 (or 0) when observing 1 (or 0). By choosing different functional forms of $m(\theta)$, the agent can make her signal correlated with the fundamental in any arbitrary way.¹⁴ Intuitively, for instance, if the agent's payoff is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, she would pay more attention to this range by making $m(\theta)$ covarying more with θ in A . This gives us a desirable account to model an agent's incentive to acquire different aspects of information.

The conditional probability $m(\cdot)$ has a natural interpretation of screening. Conditional on a fundamental θ , $m(\theta)$ is the probability of the project being screened in to get financed. In particular, it is state contingent, which captures the investor's incentive to allocate attention in screening a project. Thus, we call $m(\cdot)$ a screening rule in what follows.

We then characterize the cost of information acquisition. As in [Yang \(2012a\)](#), the amount of information conveyed by a screening rule $m(\cdot)$ is defined as the expected reduction of uncertainty through observing the signal generated, where the uncertainty associated with a distribution is measured by [Shannon \(1948\)](#)'s entropy. Formally, we use the concept of mutual information, which is defined as the difference between agents' prior entropy and expected posterior entropy:

$$\begin{aligned} I(m) &= H(\text{prior}) - H(\text{posterior}) \\ &= -g(\mathbb{E}[m(\theta)]) - (-\mathbb{E}[g(m(\theta))]) , \end{aligned}$$

where

$$g(x) = x \cdot \ln x + (1 - x) \cdot \ln(1 - x) ,$$

and the expectation operator $\mathbb{E}(\cdot)$ is with respect to θ under the probability measure Π . Denote by $M = \{m \in L(\mathbb{R}_+, \Pi) : \theta \in \mathbb{R}_+, m(\theta) \in [0, 1]\}$ the set of binary-signal information structures, and $c : M \rightarrow \mathbb{R}_+$ the cost of information. The cost is assumed to be proportional to the associated

¹³In general, an agent can choose any information structure. But an agent always prefers binary-signal information structures in binary decision problems. See [Woodford \(2008\)](#) and [Yang \(2012b\)](#) for formal discussions.

¹⁴Technically, this allows agents to choose signals drawn from any conditional distribution of the fundamental, as opposed to classic information acquisition technologies that often involve restrictions on the signals to be acquired.

mutual information:

$$c(m) = \mu \cdot I(m) ,$$

where $\mu > 0$ is the marginal cost of information acquisition per reduction of entropy.¹⁵

Built upon flexible information acquisition, the agent's problem is to choose a functional form of $m(\theta)$ to maximize her expected payoff minus the information cost. We characterize the optimal screening rule $m(\theta)$ in the following proposition. We denote $\Delta u(\theta) = u(1, \theta) - u(0, \theta)$, which is the the payoff gain of taking action 1 over action 0. We also assume that $\Pr[\Delta u(\theta) \neq 0] > 0$ to exclude the trivial case where the agent is always indifferent between the two actions. The proof is in [Yang \(2012a\)](#) (see also [Woodford, 2008](#), for an earlier treatment).

PROPOSITION 1. *Given u , Π , and μ , let $m^*(\theta) \in M$ be an optimal screening rule and*

$$\bar{\pi}^* = \mathbb{E}[m^*(\theta)]$$

be the corresponding unconditional probability of taking action 1. Then,

- i) the optimal screening rule is unique;*
- ii) there are three cases for the optimal screening rule:*

a) $\bar{\pi}^ = 1$, i.e., $\text{Prob}[m^*(\theta) = 1] = 1$ if and only if*

$$\mathbb{E}[\exp(-\mu^{-1} \cdot \Delta u(\theta))] \leq 1; \tag{2.1}$$

b) $\bar{\pi}^ = 0$, i.e., $\text{Prob}[m^*(\theta) = 0] = 1$ if and only if*

$$\mathbb{E}[\exp(\mu^{-1} \cdot \Delta u(\theta))] \leq 1;$$

c) $0 < \bar{\pi}^ < 1$ and $\text{Prob}[0 < m^*(\theta) < 1] = 1$ if and only if*

$$\mathbb{E}[\exp(\mu^{-1} \cdot \Delta u(\theta))] > 1 \text{ and } \mathbb{E}[\exp(-\mu^{-1} \cdot \Delta u(\theta))] > 1; \tag{2.2}$$

in this case, the optimal screening rule $m^(\theta)$ is determined by the equation*

$$\Delta u(\theta) = \mu \cdot (g'(m^*(\theta)) - g'(\bar{\pi}^*)) \tag{2.3}$$

for all $\theta \in \mathbb{R}_+$, where

$$g'(x) = \ln\left(\frac{x}{1-x}\right) .$$

¹⁵Although the cost $c(m)$ takes a seemingly linear form, it does not mean it is linear in information acquisition. Especially, the mutual information $I(m)$ is a non-linear functional of the screening rule $m(\cdot)$ and the prior Π , micro-founded by the information theory.

Proposition 1 fully characterizes the agent’s possible optimal decisions of information acquisition. Case a) and Case b) correspond to the scenarios where there exists an ex-ante optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios where the prior is extreme or the cost of information acquisition is sufficiently high. In contrast, Case c), the more interesting one, involves information acquisition. Especially, the optimal screening rule $m^*(\theta)$ is not constant in this case, and neither action 1 nor 0 is ex-ante optimal. This case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c) where information acquisition is involved, the agent equates the marginal benefit of information to the marginal cost of information. By doing so, the agent chooses the shape of $m^*(\theta)$ according to the shape of payoff gain $\Delta u(\theta)$ and her prior Π .¹⁶ In the next section we will see that the shape of $m^*(\theta)$ plays a critical role in characterizing how the investor screens a project.

3 Security Design

We consider the entrepreneur’s security design problem. Denote the entrepreneur’s optimal security by $s^*(\theta)$. The strategic circumstance between the entrepreneur and the investor is a dynamic Bayesian game. Concretely, the entrepreneur first designs the security, and then the investor screens the project according to the security. Hence, we apply the results in Proposition 1 to the investor’s problem, given the entrepreneur’s security, and then solve for the entrepreneur’s optimal security by backward induction. To distinguish from the general decision problem above, we denote the investor’s optimal screening rule as $m_s(\theta)$, given the security $s(\theta)$. The investor’s optimal screening rule given the entrepreneur’s optimal security $s^*(\theta)$ will be denoted by $m_s^*(\theta)$.

We formally define the equilibrium as follows.

DEFINITION 1. *Given u , Π , k and μ , the sequential equilibrium is defined as a collection of the entrepreneur’s optimal security $s^*(\theta)$ and the investor’s optimal screening rule $m_s(\theta)$ for any generic security $s(\theta)$, such that*

- i) The investor optimally acquires information at any information set: $m_s(\theta)$ is prescribed by Proposition 1,¹⁷ and*
- ii) the entrepreneur designs the optimal security:*

$$s^*(\theta) \in \arg \max_{0 \leq s(\theta) \leq \theta} \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))].$$

¹⁶See Woodford (2008), Yang (2012a,b) for more examples on this decision problem.

¹⁷The specification of belief for the investor at any generic information set is also implicitly given by Proposition 1, provided the definition of $m_s(\theta)$.

According to Proposition 1, there are three cases pertaining to the investor's behavior, given the entrepreneur's optimal security. First, the investor may optimally choose not to acquire information and accept the entrepreneur's optimal security directly. This implies that the project would be financed for sure. Second, the investor may optimally acquire some information, induced by the entrepreneur's optimal security, and then accept the entrepreneur's optimal security with positive (but less than one) probability. In this case, the project would be financed with positive (but less than one) probability from an ex-ante perspective. Third, the investor may directly reject the entrepreneur's optimal security without acquiring information, which implies that the project would not be financed. All the three cases are accommodated by the equilibrium definition. This last case, however, represents the outside option of the entrepreneur, who can always propose nothing to the investor and skip the project. Thus, it is less interesting to consider the last case, and we will focus on the first two types of equilibrium. The following lemma helps distinguish the first two cases from the last case.

LEMMA 1. *The project would be financed with positive probability in equilibrium if and only if*

$$\mathbb{E} [\exp(\mu^{-1} \cdot (\theta - k))] > 1. \quad (3.1)$$

Lemma 1 is an intuitive investment criterion. It implies that the security is more likely to be accepted by the investor, if the prior of the project's market prospect is better, if the initial investment k is smaller, or if the cost of screening μ is lower. When condition (3.1) is violated, the investor would reject the security, whatever it is.

It is interesting to note that condition (3.1) is different from the conventional expected NPV criterion, which suggests that a project should be financed when $\mathbb{E}[\theta] - k > 0$. In particular, according to condition (3.1), some projects with ex-ante negative expected NPV may be financed with positive probability. This observation is consistent with our main idea that real production depends on information production. Thanks to such dependence, investment and information acquisition are performed simultaneously, so that the conventional expected NPV criterion based on a fixed prior is generalized to a new information-adjusted one to accommodate the potential of belief updating. This also calls for a new efficiency criterion, which will be elaborated later.

The following Corollary 1 implies that the entrepreneur will never propose all the cash flow to the investor if the project would be financed. This corollary is straightforward, but we highlight it as it helps illustrate our key friction by showing that the interests of the entrepreneur and the investor are not perfectly aligned. It also helps establish some important results later. Intuitively, to retain a little bit more would still result in a finance with positive probability and give the entrepreneur a positive expected payoff.

COROLLARY 1. *When the project would be financed with positive probability, $s^*(\theta) = \theta$ is not an optimal security.*

In what follows, we assume that condition (3.1) is satisfied, and characterize the entrepreneur's optimal security, focusing on the first two types of equilibrium. As we will see, the entrepreneur's optimal securities in these two cases are different, which implies that the investor screens the project in different manners. We further show that to transfer the project at a given price is always not optimal, which also justifies the security design approach.

3.1 Optimal Security without Inducing Information Acquisition

In this subsection, we consider the case in which the entrepreneur's optimal security is directly accepted by the investor without information acquisition. In other words, the entrepreneur finds screening not worthwhile in this case and wants to design a security to deter it. Concretely, this means $Pr[m_s(\theta) = 1] = 1$. We first consider the investor's problem of screening, given the entrepreneur's security, then we characterize the optimal security.

Given a security $s(\theta)$, the investor's payoff gain by accepting the security over rejecting it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = s(\theta) - k. \quad (3.2)$$

According to Proposition 1 and conditions (2.1) and (3.2), any security $s(\theta)$ that is accepted by the investor without information acquisition must satisfy

$$\mathbb{E} [\exp(-\mu^{-1} \cdot (s(\theta) - k))] \leq 1. \quad (3.3)$$

If the left hand side of the inequality (3.3) is strictly less than one, the entrepreneur could lower $s(\theta)$ to some extent to increase her expected payoff gain, without changing the investor's incentive. Hence, condition (3.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur's problem is to choose a security $s(\theta)$ to maximize her expected payoff

$$u_E(s(\cdot)) = \mathbb{E} [\theta - s(\theta)]$$

subject to the investor's information acquisition constraint

$$\mathbb{E} [\exp(-\mu^{-1} \cdot (s(\theta) - k))] = 1,$$

and the feasibility condition $0 \leq s(\theta) \leq \theta$.¹⁸

¹⁸With this feasibility condition, the entrepreneur's individual rationality constraint $\mathbb{E} [\theta - s(\theta)] \geq 0$ is automatically satisfied, which is also true for the later case with information acquisition. This comes from the fact that the

As we would see, the entrepreneur’s optimal security in this case follows a debt. We characterize this optimal security by the following proposition, along with its graphical illustration.

PROPOSITION 2. *If the entrepreneur’s optimal security $s^*(\theta)$ induces the investor to accept the security without acquiring information in equilibrium, then it takes the form of a debt:*

$$s^*(\theta) = \min(\theta, D^*)$$

where the face value D^* is determined by

$$D^* = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu) > k,$$

in which λ is a positive constant determined in equilibrium.¹⁹

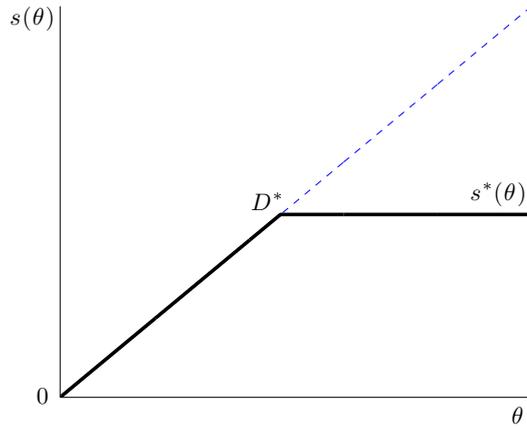


Figure 1: The Unique Optimal Security without Information Acquisition

It is intuitive to have debt as the optimal security when the entrepreneur finds it optimal not to induce information acquisition. Since screening is not worthwhile and thus the entrepreneur wants to design a security to deter it, debt is the least information sensitive one to provide the desired expected payoff to the seller. From another perspective, the optimal security renders the investor to break even between acquiring and not acquiring information. Hence, thanks to flexible information acquisition, any mean-preserving spread of the optimal security, which gives the entrepreneur the same expected payoff, would induce the investor to acquire unnecessary

entrepreneur has no money to start. It also implies that the entrepreneur always prefers to undertake the project, which is consistent with real-world practices. However, it is not correct to interpret this as that the entrepreneur would like to contract with any investor, as we do not model the competition among different investors.

¹⁹Our focus is the qualitative nature of the optimal security, instead of quantities. Thus, we do not solve for the face value in closed form, which is less tractable and does not help deliver insights. This also applies to the other optimal security discussed in a later subsection.

information. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to debt.

Debt accounts for the real-world scenarios in which some projects are financed by fixed-income financial instruments. On the one hand, when a project's prospect is clear and thus not much additional information is needed, it is often optimal to deter or mitigate investor's costly information acquisition by issuing debt, which is the least information sensitive. Interestingly, the rationale for debt as optimal in our case does not feature adverse selection, but a cost-benefit trade-off of screening. On the other hand, empirical evidence also suggests that many small businesses and less revolutionary start-ups with conventional projects rely heavily on direct bank finance (for example, [Petersen and Rajan, 1994](#); [Kerr and Nanda, 2009](#)), as opposed to more exotic financial instruments with venture capitals or buyout funds.

3.2 Optimal Security Inducing Information Acquisition

In this subsection, we characterize the entrepreneur's optimal security if it induces the investor to acquire information and accept the security with positive probability (but less than one). In other words, the entrepreneur finds screening favorable in this case and wants to design a security to incentivize it. According to [Proposition 1](#), this means $Prob[0 < m_s(\theta) < 1] = 1$.

Again, according to [Proposition 1](#) and conditions [\(2.2\)](#) and [\(3.2\)](#), any generic security $s(\theta)$ that induces the investor to acquire information must satisfy

$$\mathbb{E} [\exp (\mu^{-1} (s(\theta) - k))] > 1 \tag{3.4}$$

and

$$\mathbb{E} [\exp (-\mu^{-1} (s(\theta) - k))] > 1 , \tag{3.5}$$

Given such a security $s(\theta)$, [Proposition 1](#) and condition [\(2.3\)](#) also prescribe that the investor's optimal screening rule $m_s(\theta)$ is uniquely characterized by

$$s(\theta) - k = \mu \cdot (g'(m_s(\theta)) - g'(\bar{\pi}_s)) , \tag{3.6}$$

where

$$\bar{\pi}_s = \mathbb{E} [m_s(\theta)]$$

is the investor's unconditional probability of accepting the security and it does not depend on θ . In what follows, we also denote the unconditional probability induced by the entrepreneur's optimal security $s^*(\theta)$ by $\bar{\pi}_s^*$.

We derive the entrepreneur's optimal security by backward induction. Taking into account of

investor's response $m_s(\theta)$, the entrepreneur chooses a security $s(\theta)$ to maximize

$$u_E(s(\cdot)) = \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \quad (3.7)$$

subject to (3.4), (3.5),²⁰ (3.6), and the feasibility condition $0 \leq s(\theta) \leq \theta$.²¹

To fix idea, we first offer an intuitive roadmap to look into the optimal security and the associated optimal screening rule, highlighting their key properties. Then we give a formal proposition to characterize the optimal security and discuss its implications. The detailed derivation of the optimal security and related formal proofs are presented in the Appendix.²²

First, the investor's optimal screening rule $m_s^*(\theta)$, induced by the optimal security $s^*(\theta)$, must increase in θ . When the entrepreneur finds it optimal to induce information acquisition, she benefits from screening of the investor. Effective screening makes sense only if the investor screens in a good project and screens out a bad project; otherwise it incurs a lower social surplus. Under flexible information acquisition, this implies that $m_s^*(\theta)$ should be more likely to generate a good signal and result in a successful finance for a higher fundamental θ , while generate a bad signal and result in a rejection for a lower θ . Therefore, $m_s^*(\theta)$ should be increasing in θ . As we will see, the monotonicity of $m_s^*(\theta)$ and the shape of $s^*(\theta)$ are highly related to each other.

To induce an increasing optimal screening rule $m_s^*(\theta)$, the optimal security $s^*(\theta)$, proposed by the entrepreneur, must be increasing in θ as well, according to the first order condition of information acquisition (3.6). Intuitively, this monotonicity reflects the dependence of real production on information production: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage effective screening. As opposed to the classic security design literature that often restrict the feasible set to non-decreasing securities (for example, Innes, 1990; Nachman and Noe, 1994; DeMarzo and Duffie, 1999; DeMarzo, 2005), our prediction of an increasing optimal security without such constraints is significant.

We also argue that the non-negative constraint $s(\theta) \geq 0$ is not binding for the optimal security $s^*(\theta)$ for any $\theta > 0$. Suppose $s^*(\tilde{\theta}) = 0$ for some $\tilde{\theta} > 0$. Since $s^*(\theta)$ is increasing in θ , for all $0 \leq \theta \leq \tilde{\theta}$ we must have $s^*(\theta) = 0$. This violates the argument above that $s^*(\theta)$ must be increasing in θ . Intuitively, zero payoffs in bad states give the investor too little incentive to acquire information, which is not optimal for the entrepreneur. The security with zero payoffs in bad states looks closest to common stock, which is the least used security between entrepreneurs and investors (Kaplan and Stromberg, 2003; Kaplan and Lerner, 2010; Lerner, Leamon and Hardyman, 2012).

²⁰According to Proposition 1, both conditions (3.4) and (3.5) should not be binding for the optimal security; otherwise the investor would not acquire information.

²¹Again, the entrepreneur's individual rationality constraint $\mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \geq 0$ is automatically satisfied.

²²To facilitate understanding, the intuitive investigation of the optimal security is not organized in the same order as the proofs in the Appendix, but all the claims in the main text are guaranteed by the formal proofs.

To shot a closer look into the optimal security, it is instructive to follow a perturbation argument on the entrepreneur's security design problem, which gives the entrepreneur's first order condition. Specifically, denote by $r^*(\theta)$ the marginal contribution to the entrepreneur's expected payoff $u_E(s(\cdot))$ by any feasible perturbation to the optimal security $s^*(\theta)$.²³ As $s^*(\theta) > 0$ for any $\theta > 0$, it is intuitive to show that for any $\theta > 0$:

$$r^*(\theta) \begin{cases} = 0 & \text{if } 0 < s^*(\theta) < \theta \\ \geq 0 & \text{if } s^*(\theta) = \theta \end{cases},$$

which is further shown to be equivalent to

$$(1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} = \mu & \text{if } 0 < s^*(\theta) < \theta \\ \geq \mu & \text{if } s^*(\theta) = \theta \end{cases}, \quad (3.8)$$

where w^* is a constant determined in equilibrium.

We now argue that the optimal security $s^*(\theta)$ follows the 45° degree line in bad states and then increases in θ in good states with a smaller slope. That is, the residual of the optimal security, $\theta - s^*(\theta)$, increases in θ as well in good states. Due to the entrepreneur's first order condition (3.8) and the monotonicity of $m_s^*(\theta)$, if $s^*(\hat{\theta}) = \hat{\theta}$ for some $\hat{\theta} > 0$, it must be $s^*(\theta) = \theta$ for any $0 < \theta < \hat{\theta}$. Similarly, if $s^*(\hat{\theta}) < \hat{\theta}$ for some $\hat{\theta} > 0$, it must be $s^*(\theta) < \theta$ for any $\theta > \hat{\theta}$. In addition, Corollary 1 rules out $s^*(\theta) = \theta$ for all $\theta > 0$ as an optimal security. Thus, since $s^*(\theta)$ is increasing in θ , the limited liability constraint can only be binding in bad states.²⁴ Importantly, according to condition (3.8) and again the monotonicity of $m_s^*(\theta)$, when the limited liability constraint is not binding in good states, not only $s^*(\theta)$ but also $\theta - s^*(\theta)$ is increasing in θ . In other words, $s^*(\theta)$ is dual monotone when it deviates from the the 45° degree line in good states.

The shape of the optimal security $s^*(\theta)$ reflects the economy's friction in a clear way. Recall that the monotonicity of $s^*(\theta)$ reflects the dependence of information and real production. The monotonicity of $\theta - s^*(\theta)$ however reflects their separation: the entrepreneur wants to retain as much as possible even if she needs the investor to screen the project. Specifically, the area between $s^*(\theta)$ and the 45° degree line not only captures the entrepreneur's retained benefit, but also reflects the degree to which the allocation of resources is inefficient. This is intuitive: the dependence renders the investor to get all the resources but the separation prevents the entrepreneur from proposing such a deal, as shown in Corollary 1. The competition of the two is alleviated in a least inefficient way: to reward the investor more but also retain more in better states, which is also

²³Formally, $r^*(\theta)$ is the Frechet derivative, the functional derivative used in the calculus of variations, of $u_E(s(\cdot))$ at $s^*(\theta)$. It is analogical to the commonly used derivative of a real-valued function of a single real variable but generalized to accommodate functions on Banach spaces.

²⁴In the proofs we show that the limited liability constraint must be binding for some states $(0, \hat{\theta})$ with $\hat{\theta} > 0$.

the best way for the entrepreneur to encourage screening. In this sense, our prediction of the dual monotonicity comes endogenously from the friction of the economy, rather than by assumption like existing literature (for example, [Biais and Mariotti, 2005](#)).

The following proposition characterizes the optimal security $s^*(\theta)$ that induces the investor to acquire information. We interpret it as a participating preferred stock, according to its shape.

PROPOSITION 3. *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it takes the following form of a participating preferred stock with a face value $\hat{\theta} > 0$:*

$$s^*(\theta) = \begin{cases} \theta & \text{if } 0 \leq \theta \leq \hat{\theta} \\ \hat{s}(\theta) & \text{if } \theta > \hat{\theta} \end{cases},$$

where $\hat{\theta}$ is determined in equilibrium and the unconstrained part $\hat{s}(\theta)$ satisfies:

i) $\hat{\theta} < \hat{s}(\theta) < \theta$;

ii) $0 < d\hat{s}(\theta)/d\theta < 1$.²⁵

Finally, the corresponding optimal screening rule satisfies $dm_s^*(\theta)/d\theta > 0$.

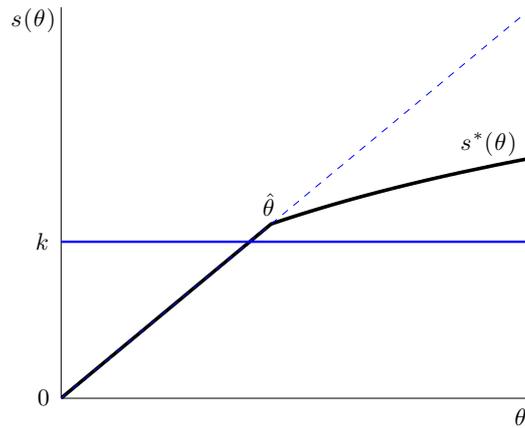


Figure 2: The Unique Optimal Security with Information Acquisition

Proposition 3 offers a clear prediction on the entrepreneur's optimal security when screening is favorable. It is closest to participating convertible preferred stock, $d\hat{s}(\theta)/d\theta$ as the converting rate, which grants the holder a right to receive both the face value and their equity participation as if it was converted, in the real-world event of a sale or liquidation. This is consistent with the empirical evidence of venture contracts, in which convertible preferred stocks account for 79.8% of total contracts used, and 48.2% of them are participating ([Kaplan and Stromberg, 2003](#)). It also fits in line with earlier evidence ([Sahlman, 1990](#); [Bergemann and Hege, 1998](#); [Gompers, 1999](#))

²⁵In the appendix, we have provided the specific implicit function that determines $d\hat{s}(\theta)/d\theta$.

on the popularity of participating convertible preferred stock in financing new projects. This prediction cannot be achieved by classic models that are silent on the screening role of investors. In particular, we are able to work with arbitrary securities on continuous states with arbitrary distributions, thanks to flexible information acquisition. For simplicity, in what follows we will refer to the optimal security in this case as convertible preferred stock.

It is interesting to contrast the results here to Yang (2012a), where the optimal security is always debt. As the seller's asset is already in-place in Yang (2012a), information acquisition is socially wasteful in that case. Especially, the information acquired by the buyer makes herself better off at the expense of the seller through endogenous adverse selection. As a result, the seller designs the debt to optimally discourage harmful information acquisition. In contrast, in our production economy, the entrepreneur and the investor are jointly exposed to the cash flow of the project if the investor accepts the security, and not so if the security is rejected. In this case, information acquisition could be socially valuable and the conflicting interests of the two parties could be partly reconciled. Therefore, the entrepreneur may design a security to encourage the investor to acquire information that helps screen the project in favor of herself. This comparison sheds lights on the difference between a production economy (or a primary financial market) and an exchange economy (or a secondary financial market). In a production economy (or a primary financial market), real production depends on information production; while in an exchange economy (or a secondary financial market) information acquisition only helps reallocate existing resources. Therefore, when financing involves information acquisition, different economies (or financial markets) may require different forms of optimal securities, because the incentives of information acquisition and the underlying values of information are different accordingly.

For our prediction of convertible preferred stock, the following corollary further supports its consistency with the empirical evidence. It speaks to the multiple, defined as the ratio of the face value $\hat{\theta}$ of convertible preferred stock to the investor's initial investment k . It is viewed as a key characteristic of convertible preferred stock, analogous to the role of returns for other securities.

COROLLARY 2. *The multiple of convertible preferred stock, as the optimal security $s^*(\theta)$ that induces information acquisition in equilibrium, is greater than one. In other words, $\hat{\theta} > k$.*

Corollary 2 is consistent with the empirical documents (Kaplan and Stromberg, 2003; Lerner, Leamon and Hardyman, 2012). In practice, the multiple is always greater than one to ensure the investors' keen interest in converting when they exit.²⁶ In our context, this property comes from the idea that the entrepreneur should compensate the investor for not only the physical investment but also the screening service.

²⁶Although we do not model the exiting of investors, it is incorporated in the distribution of the cash flow.

3.3 Project Transfer

This subsection considers the potential for transferring the project at a fixed non-negative price and shows that it is not optimal. Equivalently, it also represents a scenario where the entrepreneur works for the investor as a worker and gets a fixed wage payment, as there is no moral hazard. According to [Rajan \(2012\)](#), entrepreneurs' human capital is often inalienable because of the high differentiation of young firms, which justifies our security design approach. Here, we further argue that, even if the project is transferrable, the entrepreneur still finds such a transfer not optimal.

The key to understand the idea is to view project transfer as a feasible security, and show that it is not optimal. When the entrepreneur proposes a project transfer to the investor at a fixed price $p \geq 0$, it is equivalent for her to propose a security $s(\theta) = \theta - p$ without the non-negative constraint $s(\theta) \geq 0$. To see why, if the investor accept the offer of transfer and undertake the project, she gets the entire cash flow θ and pay p as an upfront cost. This interpretation allows us to analyze project transfer in our security design framework.

To see why the equivalent security $s(\theta) = \theta - p$ is feasible but not optimal, it is important to observe that the non-negative constraint $s(\theta) \geq 0$ is binding in neither case of security design. Hence, it is equivalent to consider a larger set of feasible securities, which is still restricted by the limited liability constraint $s(\theta) \leq \theta$ but allows negative payoffs to the investor. As debt and convertible preferred stock are still the only two optimal securities in this generalized problem, and $s(\theta) = \theta - p$ representing project transfer is feasible, we conclude that project transfer is not optimal to the entrepreneur for any transfer price p . Intuitively, project transfer is not optimal because it does not follow the least costly way to compensate the investor, no matter whether information acquisition is induced.

PROPOSITION 4. *When the project can be financed with positive probability, to transfer the project at a fixed price $p \geq 0$ is not optimal for the entrepreneur.*

The timing and the friction of our production economy are important for Proposition 4. Consider an alternative timing in which the investor can only acquire information after the deal of transfer. The friction of the production economy is no longer present, because real production and information production are both performed by the investor after the transfer. Hence, under this timing, project transfer is optimal to the entrepreneur through setting a price that is equivalent to the expected profit of the investor. In this case, the entrepreneur's bargaining power is too strong in the sense that she can prevent the investor from acquiring information when proposing the transfer deal, which essentially removes the friction of the economy.

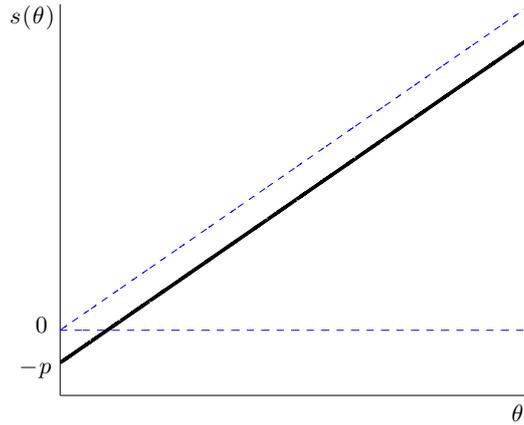


Figure 3: Project Transfer as a Security

4 Pecking Orders of Optimal Securities

Having characterized the optimal securities with and without inducing screening, we take them together and determine the optimal security given characteristics of the production economy. This helps generalize the pecking order theory (Myers and Majluf, 1984) in the presence of production and information acquisition: the entrepreneur chooses different optimal securities and thus different capital structures in different circumstances. The new pecking orders help unify debt and convertible preferred stock, which are often viewed as two distinct securities in many aspects. Our approach also bridges the security design literature and the classic pecking order theory, since our new pecking orders come from security design over a general space of feasible securities, rather than a set of commonly observed ones like debt and equity.

Our new pecking orders are presented against two benchmarks: the conventional NPV criterion and a new efficiency criterion. The first benchmark offers an intuitive look into the different applicability of the two optimal securities, and highlights two different roles of screening: screening in and out. The second allows us to reveal the relationship between the friction of the production economy and the optimal security in a more fundamental manner. Whether convertible preferred stock or debt is optimal depends on whether or not the entrepreneur wants to encourage screening, which further depends on the different extent of dependence of real production on information production. If the dependence is strong, the friction of the economy is severe. Screening is thus more valuable in this case, so that the entrepreneur finds it more worthwhile to induce screening and proposes convertible preferred stock. Otherwise, the friction is not severe and inducing costly screening is not necessary for the entrepreneur, so that debt is optimal.

4.1 An NPV Benchmark for Pecking Order

We first benchmark our new pecking order to the conventional expected NPV criterion.

PROPOSITION 5. *When the project is financed with positive probability:*

- i). If $\mathbb{E}[\theta] \leq k$, the optimal security $s^*(\theta)$ is convertible preferred stock; or*
- ii). If $\mathbb{E}[\theta] > k$, $s^*(\theta)$ may be either convertible preferred stock or debt.²⁷*

Intuitively, a negative NPV project may only be financed by convertible preferred stock: only through screening could it be potentially found good and worth financing. This is also consistent with the traditional NPV criterion that a negative NPV project can never be financed by debt with a given and fixed belief.

Interestingly, convertible preferred stock may be optimal for financing both negative and positive NPV projects, but the underlying mechanisms of screening are subtly different. In both cases, the dependence of real production on information production is strong.

When the project has a zero or negative NPV, convertible preferred stock is used to encourage the investor to screen in a potentially good project. In this case, the investor will never finance the project if she is unable to screen it, because it only incurs an expected loss even if the entrepreneur promises all the cash flow. Thus, if it would be financed, the only way is to use convertible preferred stock to encourage screening. This implies that the dependence of real production on information production is strong due to the relatively poor prior, and thus the friction is severe. When the investor acquires information, she may expect either a good signal that leads to a successful deal or a bad signal that results in a rejection, but the ex-ante probability of finance becomes positive since a potentially good project can be screened in. Hence, the entrepreneur is better off by proposing convertible preferred stock.

In contrast, when the project has a positive NPV, convertible preferred stock may still be used, but to encourage the investor to screen out a potentially bad project. Here, the dependence of real production on information production is still strong due to a relatively mediocre prior. In the status quo where the investor is unable to screen the project, the entrepreneur can finance the positive NPV project for sure by proposing debt with a high face value. However, when the investor can acquire information, the entrepreneur might find such a sure finance too expensive or even impossible, because by doing so she retains too little for herself. Instead, the entrepreneur could retain more by offering a less generous convertible preferred stock and invite the investor to screen the project. Although this results in a probability of finance less than one, the entrepreneur's total expected profit is higher since a potentially bad project may be screened

²⁷We do not give explicit conditions to distinguish between the two optimal securities when the NPV is positive. Technically, doing this requires a detailed specification of the prior, which does not help deliver insights in general. Reassuring this, we have a full dichotomy of the optimal securities with the efficiency benchmark discussed later.

out, which justifies convertible preferred stock as optimal.

Finally, debt may be optimal for some circumstances with a positive NPV project. When the prior is sufficiently good, the dependence of real production on information production is weak, and thus the benefit from screening does not justify the cost. Hence, it is optimal for the entrepreneur to propose debt to deter costly screening while still retain enough for herself.

4.2 An Efficiency Benchmark for Pecking Order

We then benchmark our pecking order to a more fundamental efficiency criterion. To understand how the optimal security evolves with the severity of the friction, we consider a frictionless centralized economy in which real production and information production are aligned. We define an efficiency criterion with help of this centralized economy. We show that, if and only if the friction in the decentralized economy is not severe in the sense that an optimal security can achieve efficiency, the optimal security is debt and screening is not induced in equilibrium. If and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency, the optimal security is convertible preferred stock and screening is induced. This dichotomy again highlights the close connection among the shape of the optimal security, the role of screening, and the extent of the friction in the production economy.

We start by defining the expected social surplus and the efficiency criterion. The expected surplus of our decentralized economy is the difference between the expected profit of the project and the cost of screening, both of which are functions of the screening rule. Thus, an optimal security in the decentralized economy achieves efficiency if the associated optimal screening rule maximizes expected social surplus in equilibrium.

DEFINITION 2. *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule $m_s^*(\theta)$ satisfies:*

$$m_s^*(\theta) \in \arg \max_{0 \leq m(\theta) \leq 1} \mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)). \quad (4.9)$$

To facilitate discussion, we characterize a frictionless centralized economy that helps benchmark the friction in the corresponding decentralized economy. In the centralized economy, u , Θ , Π , k and μ are given as the same. However, we suppose the entrepreneur has enough initial wealth and can also screen the project. Thus, real production and information production are aligned. In this economy, security design is irrelevant. The entrepreneur's problem is to decide whether to undertake the project directly, to screen it, or to skip it directly. The entrepreneur's payoff gain

by undertaking the project over skipping it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k.$$

We denote the screening rule in the centralized economy by $m_c(\theta)$ and the optimal one by $m_c^*(\theta)$. Thus, the entrepreneur's problem in the centralized economy is

$$\max_{0 \leq m_c(\theta) \leq 1} \mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\theta)). \quad (4.10)$$

By construction, the entrepreneur's objective (4.10) in the centralized economy is just the expected social surplus in the decentralized economy. It is convenient to work with the centralized economy to analyze the efficiency of equilibria in the corresponding decentralized economy.

Since the optimal screening rules are unique for both the centralized and decentralized economies, efficiency is achieved if and only if information is acquired in the same manner in both the decentralized economy and the centralized economy.

LEMMA 2. *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule $m_s^*(\theta)$ satisfies*

$$\text{Prob}[m_s^*(\theta) = m_c^*(\theta)] = 1,$$

where $m_c^*(\theta)$ is the optimal screening rule in the corresponding centralized economy.

The efficiency criterion in Lemma 2 highlights the role of screening in the production economy and provides a natural measure of the severity of friction in the decentralized economy. Fundamentally, we may view the expected social surplus (4.10) or (4.9) in our production economy as a production function, information characterized by the screening rule $m_c(\theta)$ or $m(\theta)$ as the sole input. This again fits in line with our idea that real production depends on information production. Consequently, efficiency is achieved if and only if the optimal security in the decentralized economy delivers the same equilibrium allocation of input as what the centralized economy does. If the optimal security achieves this job, the friction in the decentralized economy is not severe as it can be effectively removed by the optimal security. Otherwise, the friction is severe since it cannot be completely removed even if an optimal security is used.

With the efficiency criterion in Definition 2 and Lemma 2, we are able to characterize our pecking order of optimal securities as follows.

PROPOSITION 6. *In the decentralized production economy, when the project is financed with positive probability:*

- i) The optimal security $s^*(\theta)$ is debt if and only if the friction of the decentralized economy is not severe in the sense that an optimal security achieves efficiency; and*
- ii) $s^*(\theta)$ is convertible preferred stock if and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency.*

This pecking order against the efficiency criterion is important not only because it distinguishes between the two optimal securities given characteristics of the production economy, but also because it wraps up the roles of optimal securities and screening in reducing the friction of our economy. In the decentralized economy, real production is performed by the entrepreneur while information production by the investor. The separation is always present and unchanged in spite of different exogenous characteristics of the economy. Hence, the severity of friction is reflected by the extent to which real production depends on information production. If the friction is severe, the dependence is strong, which makes screening worthwhile and thus renders convertible preferred stock as optimal. Similarly, if the friction is not severe, the dependence is weak. As a consequence, the benefit of screening does not justify its cost and thus debt is optimal.

Our pecking orders shed new lights on unifying empirical evidence. Debt financing is popular for conventional projects and for investors who have less expertise in screening, which represent the cases where the friction is not severe. Instead, financing with convertible preferred stock is common for innovative projects that need more screening and for investors who are more capable of doing that, which represents the cases where the friction is severe.

5 Numerical Comparative Statics of the Optimal Security

To build more intuitions, we numerically perform comparative statics of the shape of optimal securities with respect to three empirical dimensions: the profitability of the project, the uncertainty of the project, and the cost of screening.²⁸ When the parameters vary, the role of screening changes and the way through which the entrepreneur incentivizes screening changes accordingly, which leads to different shapes of optimal securities. This unifies all the comparative statics.

5.1 Profitability of Project

First, we consider the effects of varying the investment k on the shape of the optimal security $s^*(\theta)$. We fix the project's market prospect, the prior distribution of the fundamental θ , as well as the cost of screening, μ . Thus, a decrease in k implies that the project is more profitable ex-ante.

²⁸We are not aware of any analytical comparative statics pertaining to general functionals. An analytical comparative statics requires a total order, which is not applicable for our security space. Even for some characteristics of the optimal security that have orders, for instance, the face value, analytical comparative statics are not achievable. Thus, we rely on numerical results to deliver intuitions and leave analytical work to future research. Numerical analysis in our framework is tractable but already technically intensive, and the codes are available upon request.

We show the results in Figure 4. The prior of the fundamental θ is generated as follows. We take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate and normalize this distribution to the interval $[0, 1]$. The screening cost μ is fixed at 0.2. The investment k takes three increasing values: 0.4, 0.475, and 0.525. The optimal security is debt when $k = 0.4$, and is convertible preferred stock for another two projects when k takes larger values, one with positive NPV and the other negative. In particular, the face value $\hat{\theta}$ and the converting rate $d\hat{s}(\theta)/d\theta$ of the convertible preferred stock are both increasing in k .

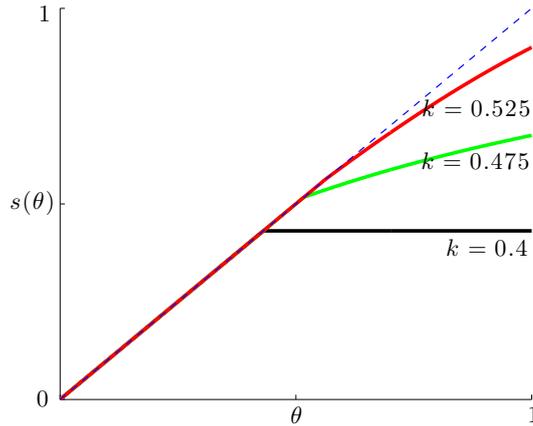


Figure 4: Change of Investment with $\mathbb{E}[\theta] = 0.5, \mu = 0.2$

The comparative statics with respect to the profitability of the project serves as a detailed illustration of Proposition 5. When the project is profitable ex-ante, the friction is not severe, and thus it is financed by debt without inducing screening. When the project looks mediocre in terms of its profitability but still has a positive NPV, the friction becomes severe, and information acquisition becomes worthwhile to screen out a potentially bad project. When the project is not profitable in the sense that its NPV is negative, the friction is more severe. The only way to finance it is to propose convertible preferred stock and to expect a potentially good project being screened in. Especially, for the last project, screening is more valuable, and thus the entrepreneur is willing to compensate the investor more generously to induce more effective screening.

5.2 Uncertainty of Project

We consider the effects of changing the project's uncertainty on the optimal security $s^*(\theta)$. Specifically, we consider prior distributions of the fundamental θ with the same mean, ranked by second order stochastic dominance.²⁹ We also fix the investment k as well as the cost of screening μ .

²⁹There are other ways to measure the project's uncertainty. For comparative statics, our idea is to find a partial order of uncertainty over the space of distributions, while to keep the project's ex-ante NPV constant. Thus, second order stochastic dominance is a natural choice.

Note that, the effect of varying uncertainty cannot be accounted by any argument involving risks, because both the entrepreneur and the investor in our context are risk neutral. Instead, we still focus on the friction and the role of screening to explain these effects.

Interestingly, the comparative statics with respect to the project's uncertainty depend on the sign of its ex-ante NPV. As highlighted in Proposition 5, the role of screening is different when the projects have different signs of NPV. This further leads to different patterns of comparative statics when the uncertainty varies.

First, we consider projects with a fixed positive NPV and increasing uncertainty. The results are in Figure 5, the left panel illustrating the priors of the fundamental θ and the right panel the evolution of the optimal security. The investment is $k = 0.4$, and the cost of screening is $\mu = 0.2$. For the priors, we take normal distributions with mean 0.5 and standard deviations 0.125 and 0.25, and then truncate and normalize them to the interval $[0, 1]$. We also construct a third distribution, in which the project is so uncertain that the cash flow has more chances to take extreme values in $[0, 1]$. As shown, the optimal security is debt when the project is the least uncertain. Convertible preferred stock applies as optimal for financing the other two more uncertain projects, while the face value $\hat{\theta}$ and the converting rate $d\hat{s}(\theta)/d\theta$ are both increasing in uncertainty.

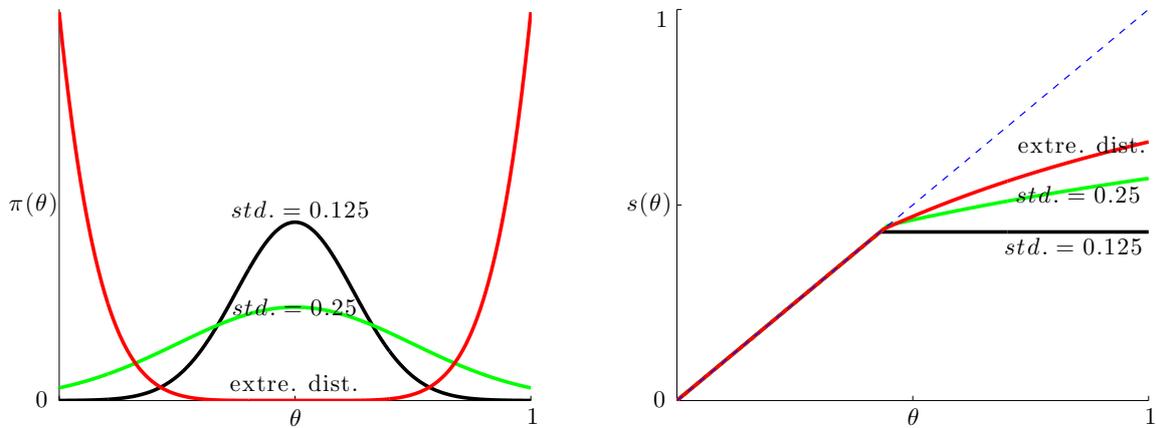


Figure 5: Change of Uncertainty: $k = 0.4 < \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case demonstrate how varying uncertainty affects the role of screening out a potentially bad project, given a positive ex-ante NPV. When the the project is the least uncertain, it is least likely to be bad, which implies that screening out is least relevant and thus debt financing is optimal. When the uncertainty becomes greater, the project is more likely to be bad, and screening out becomes more valuable. Hence, the entrepreneur finds it optimal to propose a more generous convertible preferred stock to induce screening out.

Then we consider projects with a fixed negative NPV. We focus on the projects that may be financed with positive probability, thanks to screening in through convertible preferred stock. The results are in Figure 6. The investment is $k = 0.525$ and the cost of screening is $\mu = 0.2$. The priors are generated as we did in Figure 5. In this case, both the face value $\hat{\theta}$ and the converting rate $d\hat{s}(\theta)/d\theta$ of convertible preferred stock are decreasing in uncertainty.

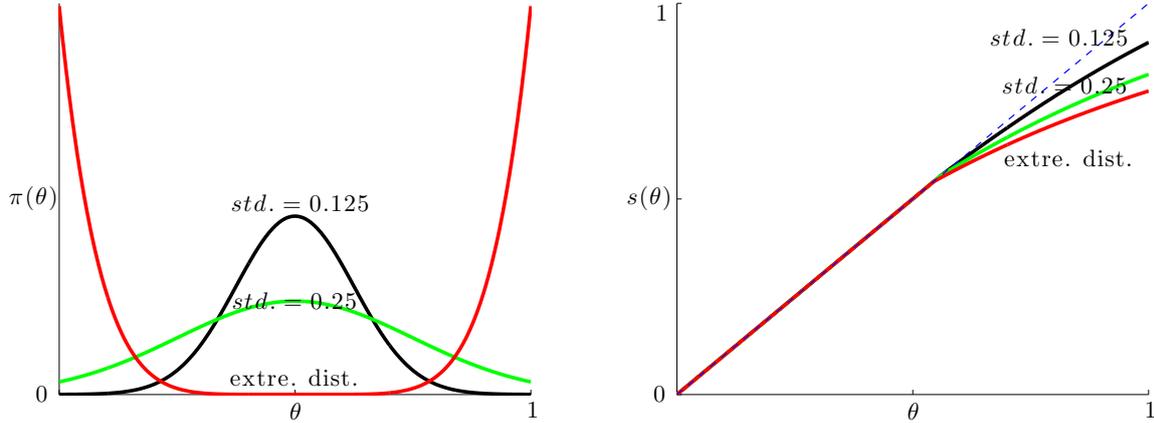


Figure 6: Change of Uncertainty: $k = 0.525 \geq \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case are also intuitive due to the role of screening. Given a negative ex-ante NPV, the investor screens in a potentially good project. In contrast to the case with a positive NPV, when the project is more uncertain, the project is instead more likely to be good. Thus, to acquire costly information to screen in a potentially good project becomes less necessary. Therefore, the entrepreneur wants to propose a less generous convertible preferred stock for less costly screening. Not surprisingly, the resulting convertible preferred stock moves away from the 45° degree line when the project is more uncertain.

5.3 The Cost of Screening

We consider the effects of changing the cost of screening μ on the optimal security $s^*(\theta)$, with the prior of the fundamental θ and the investment k fixed. Again, the comparative statics depend on the sign of the project's NPV, and fundamentally, the different roles of screening in and out.

First, we consider a positive NPV project with increasing cost of screening. The results are in Figure 7. The investment is $k = 0.4$. The prior distribution of the fundamental θ is also fixed: we take a normal distribution with mean 0.5 and standard deviation 2, and then truncate and normalize it to the interval $[0, 1]$. The cost of screening μ takes three increasing values: 0.2, 0.4, and 1. When μ is low, convertible preferred stock is optimal, with the face value $\hat{\theta}$ increasing but the converting rate $d\hat{s}(\theta)/d\theta$ decreasing in μ . When μ is 1, the highest, the optimal security is

debt with the highest face value and the degenerated converting rate 0.

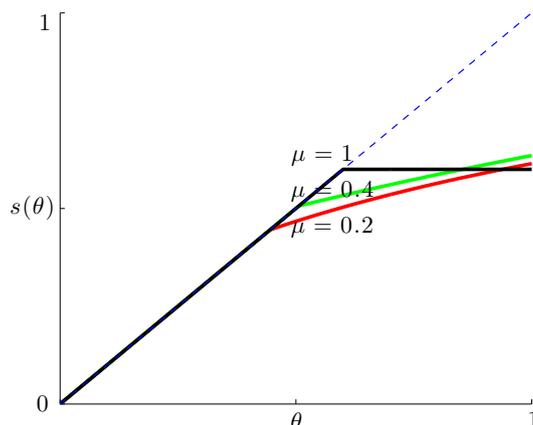


Figure 7: Change of Information Cost: $k = 0.4 < \mathbb{E}[\theta] = 0.5$

The intuition is clear, thanks to the role of screening on a positive NPV project. When the cost of screening becomes higher, it is more expensive for the entrepreneur to incentivize the investor to screen out a potentially bad project. Hence, the entrepreneur has to give up more by raising the face value of the proposed security, which makes the investor more comfortable in financing a potentially bad project. At the same time, the entrepreneur reduces the converting rate to induce less costly screening. When the cost of screening is too high, the entrepreneur finds it completely worthless to induce screening out, so that she just use debt with a sufficiently high face value to achieve a sure finance without inducing any screening.

Finally, we consider a negative NPV project with increasing cost of screening, in which case only convertible preferred stock is optimal. The results are in Figure 8. The investment is $k = 0.6$. The prior is the same as the last case: a normal distribution with mean 0.5 and standard deviation 2, and then truncated and normalized to $[0, 1]$. The cost of screening μ takes there increasing values: 0.075, 0.125, and 0.225. As shown, both the face value $\hat{\theta}$ and the converting rate $d\hat{s}(\theta)/d\theta$ of convertible preferred stock are increasing in μ .

The intuition for this case is again straightforward under our unified account of screening. When financing the project with a negative NPV, the investor acquires information to screen in a potentially good project. As screening through convertible preferred stock is the only way to finance a project with a negative ex-ante NPV, the entrepreneur has to compensate the investor more when the cost of screening is higher. This results in a more generous convertible preferred stock.

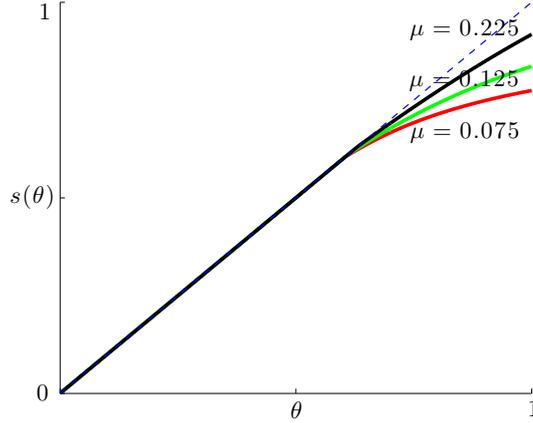


Figure 8: Change of Information Cost: $k = 0.6 \geq \mathbb{E}[\theta] = 0.5$

6 Conclusion

Highlighting a new informational friction, this paper have investigated security design in a production economy. Real production depends on information production while these two are separately performed by an entrepreneur and an investor. A new pecking order of optimal securities has been predicted: debt, which does not induce screening, is optimal when the dependence is weak and thus the friction is not severe, and convertible preferred stock, which induces screening, is optimal when the dependence is strong and thus the friction is severe. Both the optimal securities and the pecking order are supported by empirical evidence.

This paper contributes to several fronts of security design and information acquisition. Information insensitive and sensitive securities have been unified to a new but single origin: in financing different projects with different extents of friction, the investor's expertise of screening is called for in different manners, which further shapes the optimal securities. The comparison between the production economy and an exchange economy further highlights this origin. Thanks to the new concept of flexible information acquisition, we have been able to work with arbitrary securities on continuous states without any distributional assumptions.

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A Appendix

A.1 Derivation of Convertible Preferred Stock as the Optimal Security

This appendix derives the optimal security $s^*(\theta)$ when it induces information acquisition. To make the intuition clearer, we proceed by two steps.

First, we solve for an “unconstrained” optimal security in this case without the feasibility condition $0 \leq s(\theta) \leq \theta$. We denote the solution by $\widehat{s}(\theta)$. We also denote the corresponding screening rule by $\widehat{m}_s(\theta)$.³⁰ We try to highlight that the feasibility condition is a mechanical restriction on the potential securities. Importantly, it is not directly relevant to the entrepreneur’s motives in designing the securities, which are aiming to induce the investor to acquire information that is most favorable to the entrepreneur. Hence, the unconstrained optimal security $\widehat{s}(\theta)$ helps to reveal the relationship between security design and information acquisition in a clearer manner. After that, we resume the feasibility condition and characterize the exact optimal security $s^*(\theta)$. This two-step approach streamlines our presentation and makes the intuition clearer.

For the unconstrained optimal security $\widehat{s}(\theta)$ in the case with information acquisition, we have the following lemma.

LEMMA 3. *In an equilibrium with information acquisition, the unconstrained optimal security $\widehat{s}(\theta)$ and its corresponding screening rule $\widehat{m}_s(\theta)$ are determined by*

$$\widehat{s}(\theta) - k = \mu \cdot (g'(\widehat{m}_s(\theta)) - g'(\bar{\pi}_s^*)) , \quad (\text{A.1})$$

where

$$\bar{\pi}_s^* = \mathbb{E}[m_s^*(\theta)] ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu , \quad (\text{A.2})$$

where

$$w^* = \mathbb{E} \left[(\theta - s^*(\theta)) \frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \left(1 - \mathbb{E} \left[\frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \right)^{-1} ,$$

in which \bar{p}_s^* and w^* are two constants determined in equilibrium.

Lemma 3 exhibits the relationship between the unconstrained optimal security and the corresponding screening rule. Recall Proposition 1, condition (A.1) specifies how the investor responses to the unconstrained optimal security $\widehat{s}(\theta)$ by adjusting her screening rule $\widehat{m}_s(\theta)$. On the other hand, condition (A.2) is derived from the entrepreneur’s optimization problem. It describes the

³⁰Note that, $\widehat{m}_s(\theta)$ is not the solution to the entrepreneur’s problem without the feasibility condition, but is a translation of that solution. This will be seen clearer in the statement of Lemma 3.

entrepreneur's optimal choices of cash flow across states, which is summarized by the unconstrained optimal security, given the investor's screening rule. In equilibrium, $\widehat{s}(\theta)$ and $\widehat{m}_s(\theta)$ are jointly determined. This again highlights the close relationship between security design and information acquisition in our context.

Although it is mathematically difficult to solve the system of equations (A.1) and (A.2), we are able to deliver some important analytical characteristics of the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$. In particular, we are confident enough to speak to the shape of the unconstrained optimal security $\widehat{s}(\theta)$ as well as the actual optimal security $s^*(\theta)$ by these analytical properties. The following lemma gives the key results.

LEMMA 4. *In an equilibrium with information acquisition, the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$ satisfy*

$$\frac{\partial \widehat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 > 0, \quad (\text{A.3})$$

and

$$\frac{\partial \widehat{s}(\theta)}{\partial \theta} = 1 - \widehat{m}_s(\theta) \in (0, 1). \quad (\text{A.4})$$

We have several interesting observations from Lemma 4. First, condition (A.3) implies that the unconstrained optimal screening rule $\widehat{m}_s(\theta)$ is strictly increasing. Second, condition (A.4) implies that the unconstrained optimal security $\widehat{s}(\theta)$ is also strictly increasing. These are because, according to Proposition 1, we have $Prob[0 < \widehat{m}_s(\theta) < 1] = 1$ in this case, and thus the right hand sides of (A.3) and (A.4) is positive. Last, the unconstrained optimal security $\widehat{s}(\theta)$ is strictly concave. This is because conditions (A.3) and (A.4) imply

$$\frac{\partial^2 \widehat{s}(\theta)}{\partial \theta^2} = -\mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 < 0.$$

Therefore, the unconstrained optimal security $\widehat{s}(\theta)$ is an increasing concave function of θ .

The monotonicity of the unconstrained screening rule $\widehat{m}_s(\theta)$ as shown in (A.3) is intuitive. In our context, the investor provides two different types of services to the entrepreneur. The first is the investment required to initiate the project, and the second is the information about the project's market prospect, which implies screening in a good project and screening out a bad project. In other words, a better fundamental would be more likely to generate a good signal and result in a successful finance, while a worse fundamental would to more likely to generate a bad signal and result in a rejection. As a result, the screening service makes sense only if the unconstrained optimal screening rule $\widehat{m}_s(\theta)$ is increasing.

Moreover, the monotonicity of $\widehat{m}_s(\theta)$ has two important implications on the shape of the

unconstrained optimal security $\widehat{s}(\theta)$. On the one hand, according to condition (A.1), $\widehat{s}(\theta)$ is increasing because $\widehat{m}_s(\theta)$ is increasing. This reflects the dependence of real production on information production: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage screening. On the other hand, according to condition (A.2), $\theta - \widehat{s}(\theta)$ is also increasing because $\widehat{m}_s(\theta)$ is increasing. This however reflects the separation of real production and information production: the entrepreneur wants to retain as much as possible. Again as $\widehat{m}_s(\theta)$ is increasing, the competition of the two effects above implies that the least costly way for the entrepreneur to encourage the investor to acquire information is to reward the investor more but also retain more in higher states.

Given the unconstrained optimal security $\widehat{s}(\theta)$, it is instructive to have the following lemma to illustrate the possible relative positions between the unconstrained optimal security and the feasibility constraints.

LEMMA 5. *Three possible relative positions between the unconstrained optimal security $\widehat{s}(\theta)$ and the feasibility constraints $0 \leq s(\theta) \leq \theta$ may occur in equilibrium, in the $\theta \sim s$ space:*

i) $\widehat{s}(\theta)$ intersects with the 45° line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, and does not intersect with the horizontal axis $s = 0$.

ii) $\widehat{s}(\theta)$ goes through the origin $(0, 0)$, and does not intersect with either the 45° line $s = \theta$ or the horizontal axis $s = 0$ for any $\theta \neq 0$.

iii) $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, and does not intersect with the 45° line $s = \theta$.

In the three different cases, it is easy to imagine that the actual optimal security $s^*(\theta)$ will be constrained by the feasibility condition in different ways. For example, the optimal security will be constrained by the 45° line $s = \theta$ in Case i) and by the horizontal axis $s = 0$ in Case iii). Our concern is whether the presence of the two feasibility constraints would significantly change the interplay between security design and information acquisition, and thus the resulting optimal security. Fortunately, the answer is no. As expected, the feasibility constraints are only mechanical. By imposing the feasibility conditions, we have the following characterization for the optimal security:

LEMMA 6. *In an equilibrium with information acquisition, the corresponding optimal security $s^*(\theta)$ satisfies*

$$s^*(\theta) = \begin{cases} \theta & \text{if } \widehat{s}(\theta) > \theta \\ \widehat{s}(\theta) & \text{if } 0 \leq \widehat{s}(\theta) \leq \theta \\ 0 & \text{if } \widehat{s}(\theta) < 0 \end{cases} ,$$

where $\widehat{s}(\theta)$ is the corresponding unconstrained optimal security.

Lemma 6 is instructive because it tells us how to construct an optimal security with information acquisition from its corresponding unconstrained optimal security. Concretely, the optimal security $s^*(\theta)$ will follow its corresponding unconstrained optimal security $\widehat{s}(\theta)$ when the latter is within the feasible region $0 \leq s \leq \theta$. When the unconstrained optimal security is out of the feasible region, the resulting optimal security will follow one of the feasibility constraints that is binding. The expression of Lemma 6 is fairly simple but the result is not trivial. Importantly, the presence of the feasibility constraints indeed changes the shapes of the resulting optimal securities from its unconstrained counterparts, which implies that the investor's incentives of information acquisition is also changed. Nevertheless, Lemma 6 ensures us that such change does not affect the entrepreneur's choice of the cash flow allocations in the states where the feasibility constraints are not binding. Also, in the states where the feasibility constraints are binding, Lemma 6 tells us that it is still optimal for the entrepreneur to just hit the binding constraints to exploit the investor's information advantage to the largest extent.

Therefore, we can apply Lemma 6 to the three cases of the unconstrained optimal security $\widehat{s}(\theta)$ described in Lemma 5. This gives the three potential cases of the optimal security $s^*(\theta)$, respectively.

LEMMA 7. *In an equilibrium with information acquisition, the optimal security $s^*(\theta)$ may take one of the following three forms:*

i) *When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the 45° line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, we have*

$$s^*(\theta) = \begin{cases} \theta & \text{if } 0 \leq \theta < \widehat{\theta} \\ \widehat{s}(\theta) & \text{if } \theta \geq \widehat{\theta} \end{cases} .$$

ii) *When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ goes through the origin $(0, 0)$, we have $s^*(\theta) = \widehat{s}(\theta)$ for $\theta \in \mathbb{R}_+$.*

iii) *When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, we have*

$$s^*(\theta) = \begin{cases} 0 & \text{if } 0 \leq \theta < \widetilde{\theta} \\ \widehat{s}(\theta) & \text{if } \theta \geq \widetilde{\theta} \end{cases} .$$

The three potential cases of the optimal security $s^*(\theta)$ take different shapes. Specifically, in Case i), the optimal security follows a debt in bad states but increases as a concave function in good states. In Case iii), the payment of optimal security in bad states is zero, while is an increasing and concave function in good states. Case ii) lies in between as a cut-off case, in which

the payment of optimal security is an increasing and concave function.

Having characterizing the potential cases of the optimal security $s^*(\theta)$ by its differential properties, we proceed by checking whether these three potential cases are indeed the valid solution to the entrepreneur's problem in an equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium. The following proposition tells us that only the shape in Case i) can actually sustain an equilibrium with information acquisition.

LEMMA 8. *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it must follow Case i) in Lemma 7, which corresponds to a participating convertible preferred stock with a face value $\hat{\theta}$.*

Together with the results already established, this lemma straightforwardly leads to Proposition 3. Recall that the investor can always reject the offer and enjoy her outside option, which is normalized to zero in the context, if her expected payoff is negative. The rejection of the security, however, is always sub-optimal to the entrepreneur, as long as the project has a positive expected future cash flow. As a result, the entrepreneur wants to make sure that the investor, who makes the initial investment and takes cost to acquire information, is sufficiently compensated so that she is willing to accept the security.

A.2 General Allocation of Bargaining Powers

This appendix extends our benchmark model to a more general setting which allows for arbitrary allocation of the bargaining powers between the entrepreneur and the investor in security design. Suppose a third party in the economy knows the relative bargaining powers of the entrepreneur and the investor. The entrepreneur's bargaining power in security design is $1 - \alpha$ and the investor's is α . The third party designs the security and proposes it to the investor. The investor acquire information according to the security she gets and decides whether or not to accept this security. The third party's objective function is a weighted average of the entrepreneur's and the investor's utilities. The weights represent the bargaining powers of the two, respectively. When $\alpha = 0$, this reduces to our benchmark decentralized model.

In this setting, the third-party's objective function, namely, the payoff gain is

$$u_T(s(\theta)) = \alpha \cdot (\mathbb{E}[(s(\theta) - k) \cdot m(\theta)] - \mu \cdot I(m)) + (1 - \alpha) \cdot \mathbb{E}[(\theta - s(\theta)) \cdot m(\theta)].$$

We show that, the differential equation that governs information acquisition is still as same as condition (3.6):

$$s(\theta) - k = \mu \cdot (g'(m_s(\theta)) - g'(\bar{\pi}_s)) ,$$

while the other differential equation that characterizes the optimality of the unconstrained security is given as

$$r(\theta) = (2\alpha - 1) \cdot m(\theta) + (1 - \alpha) \cdot \mu^{-1} \cdot m(\theta) \cdot (1 - m(\theta)) \cdot (\theta - s(\theta) + w).$$

We have the following two propositions to characterize the optimal security in the general setting. The proofs are similar to their counterparts in the main text so that we omit them.

PROPOSITION 7. *When $0 \leq \alpha < 1/2$ and information acquisition happens in equilibrium, the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$ satisfy*

$$\frac{d\widehat{s}(\theta)}{d\theta} = \frac{1 - \widehat{m}_s(\theta)}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} \in (0, 1)$$

and

$$\frac{d\widehat{m}_s(\theta)}{d\theta} = \frac{\mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} > 0.$$

Also, all the results from Lemma 5 to Lemma 7 and from Proposition 1 to Proposition 5 still hold.

PROPOSITION 8. *When $1/2 \leq \alpha \leq 1$, the optimal security features $s^*(\theta) = \theta$.*

Our generalized results show that, when the investor has some bargain power in security design but it is still the entrepreneur who dominates, all the qualitative results keep the same. Nevertheless, if the investor dominates, the optimal security would favor the investor to the greatest extent. The latter case looks counterintuitive at a first glance. But it is indeed consistent with our context, in which the ability of information acquisition should also be accounted as a part of the investor's bargaining power. In the benchmark model, the entrepreneur proposes the security but the investor can acquire information, which exhibits a balance of total bargaining powers. Instead, when even the role of proposing the security goes to the investor, the investor is too powerful in the sense of total bargain powers.

A.3 Proofs

This appendix provides all proofs omitted above.

PROOF OF LEMMA 1. We first prove the “only if” part.

Suppose that $\mathbb{E}[\exp(\mu^{-1}(\theta - k))] \leq 1$. According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor would reject the offer without acquiring information. Since $s(\theta) \leq \theta$, the project cannot be initiated in this case.

Then we prove the “if” part.

Let $t \in (0, 1)$. Since $\mathbb{E} [\exp(\mu^{-1}(t \cdot \theta - k))]$ is continuous in t , there exists $t < 1$ such that

$$\mathbb{E} [\exp(\mu^{-1}(t \cdot \theta - k))] > 1.$$

Hence, according to Proposition 1, the security $s_t(\theta) = t \cdot \theta$ would be accepted by the investor with positive probability. Moreover, let $m_t(\theta)$ be the corresponding screening rule. As $s_t(\theta)$ would be accepted with positive probability, $m_t(\theta)$ cannot be always zero. Hence, the entrepreneur's expected payoff is $\mathbb{E}[(1 - t) \cdot \theta \cdot m_t(\theta)]$, which is strictly positive.

Note that the security $s_t(\theta)$ is a feasible security. Hence, the optimal security $s^*(\theta)$ would also be accepted with positive probability and delivers positive expected payoff to the entrepreneur. This concludes the proof. \square

PROOF OF COROLLARY 1. The proof is straightforward following the above proof of Lemma 1. Proposing $s^*(\theta) = \theta$ gives the entrepreneur a zero payoff, while proposing $s_t(\theta) = t \cdot \theta$ constructed in the proof of Lemma 1 gives her a strictly positive expected payoff. This suggests $s^*(\theta) = \theta$ is not optimal. \square

PROOF OF PROPOSITION 2. The Lagrangian of the entrepreneur's problem is

$$\mathcal{L} = \mathbb{E} [\theta - s(\theta) + \lambda \cdot (1 - \exp(\mu^{-1} \cdot (k - s(\theta)))) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta))],$$

where λ , $\eta_1(\theta)$ and $\eta_2(\theta)$ are multipliers.

The first order condition is

$$\frac{d\mathcal{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - s(\theta))) + \eta_1(\theta) - \eta_2(\theta) = 0. \quad (\text{A.5})$$

We first consider a special case that is helpful for us to solve the optimal security. If $0 < s(\theta) < \theta$, the two feasibility conditions are not binding. Thus $\eta_1(\theta) = \eta_2(\theta) = 0$, and the first order condition is simplified as

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - s(\theta))) = 0.$$

By rearrangement, we get

$$s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu). \quad (\text{A.6})$$

We denote the right hand side of (A.6), which is irrelevant of θ , as D^* . By definition, we have $D^* > 0$. Also, it is straightforward to have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - D^*)) = 0. \quad (\text{A.7})$$

In what follows, we characterize the optimal solution $s^*(\theta)$ for different regions of θ .

First, we consider the region of $\theta > D^*$. We show that $0 < s^*(\theta) < \theta$ in this region by contradiction.

If $s^*(\theta) = \theta > D^*$, we have $\eta_1(\theta) = 0$ and $\eta_2(\theta) \geq 0$. From the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - \theta)) = \eta_2(\theta) \geq 0. \quad (\text{A.8})$$

On the other hand, as $\theta > D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - D^*)) > -1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - \theta)). \quad (\text{A.9})$$

Conditions (A.7), (A.8), and (A.9) construct a contradiction. So we must have $s^*(\theta) < \theta$ if $\theta > D^*$.

Similarly, if $s^*(\theta) = 0$, we have $\eta_1(\theta) \geq 0$ and $\eta_2(\theta) = 0$. Again from the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot k) = -\eta_1(\theta) \leq 0. \quad (\text{A.10})$$

On the other hand, as $D^* > 0$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - D^*)) < -1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot k). \quad (\text{A.11})$$

Conditions (A.7), (A.10), and (A.11) construct another contradiction. So we must have $s^*(\theta) > 0$ if $\theta > D^*$.

Therefore, we have shown that $0 < s^*(\theta) < \theta$ for $\theta > D^*$. From the discussion above for the special case, we conclude that $s^*(\theta) = D^*$ for $\theta > D^*$.

We then consider the region of $\theta < D^*$. We show that $s^*(\theta) = \theta$ in this region.

Since $s^*(\theta) \leq \theta < D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - s^*(\theta))) > -1 + \lambda \cdot \mu^{-1} \cdot \exp(\mu^{-1} \cdot (k - D^*)). \quad (\text{A.12})$$

From condition (A.7), the right hand side of this inequality (A.12) is zero. Together with the first order condition (A.5), the inequality (A.12) implies that $\eta_1(\theta) = 0$ and $\eta_2(\theta) > 0$. Therefore, we have $s^*(\theta) = \theta$ in this region.

Also, from the first order condition (A.5) and the condition (A.7), it is obvious that $s^*(D^*) = D^*$.

To sum up, the entrepreneur's optimal security without inducing the investor to acquire information features a debt with face value D^* determined by condition (A.6).

We need to check that there exists $D^* > 0$ and the corresponding multiplier $\lambda > 0$ such that

$$\mathbb{E} [\exp(-\mu^{-1} \cdot (\min(\theta, D^*) - k))] = 1, \quad (\text{A.13})$$

where D^* is determined by condition (A.6).

Consider the left hand side of condition (A.13). Clearly, it is continuous and monotonically decreasing in D^* . When D^* is sufficiently large, the left hand side of (A.13) approaches $\mathbb{E} [\exp(-\mu^{-1} \cdot (\theta - k))]$, a number less than one, which is guaranteed by the condition (3.3) as well as the feasibility condition $s(\theta) \leq \theta$. On the other hand, when $D^* = 0$, it approaches $\exp(\mu^{-1} \cdot k)$, which is strictly greater than one. Hence, there exists $D^* > 0$ such that the condition (A.13) holds.

Moreover, from the condition (A.6), we also know that D^* is continuous and monotonically increasing in λ . When λ approaches zero, D^* approaches negative infinity, while when λ approaches positive infinity, D^* approaches positive infinity as well. Hence, for any $D^* > 0$ there exists a corresponding multiplier $\lambda > 0$.

Finally, suppose $D^* \leq k$. It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction. This concludes the proof. \square

PROOF OF LEMMA 3. We derive the entrepreneur's optimal security $s^*(\theta)$ and the corresponding unconstrained optimal security $\hat{s}(\theta)$ through calculus of variations. Specifically, we characterize how the entrepreneur's expected payoff responds to the perturbation of her optimal security.

Let $s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta)$ be an arbitrary perturbation of the optimal security $s^*(\theta)$. Note that the investor's optimal screening rule $m_s(\theta)$ appears in the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7), and it is implicitly determined by the proposed security $s(\theta)$ through the functional equation (3.6). Hence, we need first characterize how $m_s(\theta)$ varies with respect to the perturbation of $s^*(\theta)$. Taking derivative with respect to α at $\alpha = 0$ for both sides of (3.6) leads to

$$\begin{aligned} \mu^{-1} \varepsilon(\theta) &= g''(m_s^*(\theta)) \cdot \left. \frac{\partial m_s(\theta)}{\partial \alpha} \right|_{\alpha=0} \\ &\quad - g''(\bar{\pi}_s^*) \cdot \mathbb{E} \left. \frac{\partial m_s(\theta)}{\partial \alpha} \right|_{\alpha=0}. \end{aligned}$$

Take expectation of both sides and we get

$$\begin{aligned} &\mathbb{E} \left[\left. \frac{\partial m_s(\theta)}{\partial \alpha} \right|_{\alpha=0} \right] \\ &= \mu^{-1} \cdot \left(1 - \mathbb{E} \left[(g''(m_s^*(\theta)))^{-1} \right] \cdot g''(\bar{\pi}_s^*) \right)^{-1} \cdot \mathbb{E} \left[(g''(m_s^*(\theta)))^{-1} \varepsilon(\theta) \right]. \end{aligned}$$

Combining the above two equations, for any perturbation $s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta)$, the investor's screening rule $m_s(\cdot)$ is characterized by

$$\begin{aligned} \left. \frac{\partial m_s(\theta)}{\partial \alpha} \right|_{\alpha=0} &= \mu^{-1} \cdot (g''(m_s^*(\theta)))^{-1} \varepsilon(\theta) \\ &+ \frac{\mu^{-1} \cdot (g''(m_s^*(\theta)))^{-1} \cdot \mathbb{E} \left[(g''(m_s^*(\theta)))^{-1} \varepsilon(\theta) \right]}{(g''(\bar{\pi}_s^*))^{-1} - \mathbb{E} \left[(g''(m_s^*(\theta)))^{-1} \right]} . \end{aligned} \quad (\text{A.14})$$

We interpret condition (A.14) briefly. The first term of the right hand side of (A.14) is the investor's local response to $\varepsilon(\theta)$. It is of the same sign as the perturbation $\varepsilon(\theta)$. When the repayment of the security increases at state θ , the investor is more likely to accept the security at this state. The second term measures the investor's average response to perturbation $\varepsilon(\theta)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen's inequality. As a result, if the perturbation increases the investor's repayment on average over all states, she is more likely to accept the security as well.

Now we can calculate the variation of the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7). Taking derivative of $u_E(s(\cdot))$ with respect to α at $\alpha = 0$ leads to

$$\left. \frac{\partial u_E(s(\cdot))}{\partial \alpha} \right|_{\alpha=0} = \mathbb{E} \left[\left. \frac{\partial m_s(\theta)}{\partial \alpha} \right|_{\alpha=0} (\theta - s(\theta)) \right] - \mathbb{E} [m_s^*(\theta) \cdot \varepsilon(\theta)] . \quad (\text{A.15})$$

Substitute (A.14) into (A.15) and we get

$$\left. \frac{\partial u_E(s(\cdot))}{\partial \alpha} \right|_{\alpha=0} = \mathbb{E} [r(\theta) \cdot \varepsilon(\theta)] , \quad (\text{A.16})$$

where

$$r(\theta) = -m_s^*(\theta) + \mu^{-1} \cdot (g''(m_s^*(\theta)))^{-1} \cdot (\theta - s^*(\theta) + w^*) \quad (\text{A.17})$$

and

$$w^* = \mathbb{E} \left[(\theta - s^*(\theta)) \frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \left(1 - \mathbb{E} \left[\frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \right)^{-1} .$$

Note that w^* is a constant that does not depend on θ and will be endogenously determined in the equilibrium. Besides, $r(\theta)$ is the Frechet derivative of the entrepreneur's expected payoff $u_E(s(\cdot))$ at $s^*(\theta)$, which measures the marginal contribution of any perturbation to the entrepreneur's expected payoff when the security is optimal. Specifically, the first term of (A.17) is the direct contribution of perturbing $s^*(\theta)$ disregarding the variation of $m_s^*(\theta)$, and the second term measures the indirect contribution through the variation of $m_s^*(\theta)$. This Frechet derivative $r(\theta)$ plays an important role in shaping the entrepreneur's optimal security.

To further characterize the optimal security, we discuss the Frechet derivative $r(\theta)$ in detail.

Recall that the optimal security would be restricted by the feasibility condition $0 \leq s^*(\theta) \leq \theta$.

Let

$$A_0 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = 0\} ,$$

$$A_1 = \{\theta \in \Theta : \theta \neq 0, 0 < s^*(\theta) < \theta\} ,$$

and

$$A_2 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = \theta\} .$$

Clearly, $\{A_0, A_1, A_2\}$ is a partition of $\Theta \setminus \{0\}$. Since $s^*(\theta)$ is the optimal security, we have

$$\left. \frac{\partial u_E(s(\cdot))}{\partial \alpha} \right|_{\alpha=0} \leq 0$$

for any feasible perturbation $\varepsilon(\theta)$.³¹ Hence, condition (A.16) implies

$$r(\theta) \begin{cases} \leq 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geq 0 & \text{if } \theta \in A_2 \end{cases} . \quad (\text{A.18})$$

According to Proposition 1, when the optimal security $s^*(\theta)$ induces the investor to acquire information, we have $0 < m_s^*(\theta) < 1$ for all $\theta \in \Theta$. Hence, condition (A.18) can be rearranged as

$$\frac{r(\theta)}{m_s^*(\theta)} = -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leq 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geq 0 & \text{if } \theta \in A_2 \end{cases} . \quad (\text{A.19})$$

Recall condition (3.6), given the optimal security $s^*(\theta)$, the investor's optimal screening rule $m_s^*(\theta)$ is characterized by

$$s^*(\theta) - k = \mu \cdot (g'(m_s^*(\theta)) - g'(\bar{\pi}_s^*)) , \quad (\text{A.20})$$

where

$$\bar{\pi}_s^* = \mathbb{E}[m_s^*(\theta)]$$

is the investor's unconditional probability of accepting the optimal security $s^*(\theta)$. Conditions (A.19) and (A.20) as a system of functional equations jointly determine the optimal security $s^*(\theta)$ when it induces the investor's information acquisition.

Finally, when we focus on the unconstrained optimal security $\widehat{s}(\theta)$, note that it would not be

³¹A perturbation $\varepsilon(\theta)$ is feasible with respect to $s^*(\theta)$ if there exists $\alpha > 0$ such that for any $\theta \in \Theta$, $s^*(\theta) + \alpha \cdot \varepsilon(\theta) \in [0, \theta]$.

restricted by the feasibility condition. Hence, the corresponding Frechet derivative $r(\theta)$ would be always zero at the optimum. On the other hand, the investor's optimal screening rule would not be affected. As a result, the conditions (A.20) and (A.19) become

$$\widehat{s}(\theta) - k = \mu \cdot (g'(\widehat{m}_s(\theta)) - g'(\bar{\pi}_s^*)) ,$$

where

$$\bar{p}_s^* = \mathbb{E}[m_s^*(\theta)] ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu ,$$

where

$$w^* = \mathbb{E} \left[(\theta - s^*(\theta)) \frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \left(1 - \mathbb{E} \left[\frac{g''(\bar{\pi}_s^*)}{g''(m_s^*(\theta))} \right] \right)^{-1} ,$$

in which \bar{p}_s^* and w^* are two constants that do not depend on θ . This concludes the proof. \square

PROOF OF LEMMA 4. From Lemma 3, $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies the two differential equations (A.1) and (A.2). By condition (A.2), we get

$$\widehat{m}_s(\theta) = 1 - \frac{\mu}{\theta - \widehat{s}(\theta) + w^*} . \quad (\text{A.21})$$

Substituting (A.21) into (A.1) leads to

$$\mu^{-1} (\widehat{s}(\theta) - k) = g' \left(\frac{\mu}{\theta - \widehat{s}(\theta) + w^*} \right) - g'(\bar{\pi}_s^*) .$$

Taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned} \mu^{-1} \cdot \frac{d\widehat{s}(\theta)}{d\theta} &= g''(\widehat{m}_s(\theta)) \cdot \frac{d\widehat{m}_s(\theta)}{d\theta} \\ &= g''(\widehat{m}_s(\theta)) \cdot \frac{\mu \cdot \left(1 - \frac{d\widehat{s}(\theta)}{d\theta}\right)}{(\theta - \widehat{s}(\theta) + w^*)^2} \\ &= \frac{1 - \frac{d\widehat{s}(\theta)}{d\theta}}{\theta - \widehat{s}(\theta) + w^* - \mu} , \end{aligned}$$

where we use

$$g''(x) = \frac{1}{x(1-x)}$$

while deriving the third equality. Manipulating the above equation we get

$$\begin{aligned}\frac{d\widehat{s}(\theta)}{d\theta} &= \frac{\mu}{\theta - \widehat{s}(\theta) + w^*} \\ &= 1 - \widehat{m}_s(\theta),\end{aligned}$$

where the last equality follows (A.21).

Again, taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}\mu^{-1} \cdot \frac{d\widehat{s}(\theta)}{d\theta} &= g''(\widehat{m}_s(\theta)) \cdot \frac{d\widehat{m}_s(\theta)}{d\theta} \\ &= \frac{1}{\widehat{m}_s(\theta)(1 - \widehat{m}_s(\theta))} \cdot \frac{d\widehat{m}_s(\theta)}{d\theta}.\end{aligned}$$

Hence

$$\begin{aligned}\frac{d\widehat{m}_s(\theta)}{d\theta} &= \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta)) \cdot \frac{d\widehat{s}(\theta)}{d\theta} \\ &= \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2.\end{aligned}$$

This completes the proof. \square

PROOF OF LEMMA 5. From Lemma 4, it is easy to see that the slope of $\widehat{s}(\theta)$ is always less than one. Hence, Lemma 5 is straightforward. \square

PROOF OF LEMMA 6. We proceed by discussing three cases.

Case 1: We show that $\widehat{s}(\theta) > \theta$ would imply $s^*(\theta) = \theta$.

Suppose $s^*(\theta) < \theta$. Then we have $s^*(\theta) < \widehat{s}(\theta)$. Since both $(s^*(\theta), m_s^*(\theta))$ and $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfy condition (3.6), we must have $m_s^*(\theta) < \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\ &> -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\ &= 0,\end{aligned}$$

which implies $s^*(\theta) = \theta$, a contradiction.

Note that, the logic for the inequality above is as follows. Since $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies condition (A.2), we must have $\theta - \widehat{\theta} + w^* > 0$. Hence, $\widehat{s}(\theta) > s^*(\theta)$ implies that

$$\theta - s^*(\theta) + w^* > \theta - \widehat{s}(\theta) + w^* > 0.$$

Also note that $1 - m_s^*(\theta) > 1 - \widehat{m}_s(\theta) > 0$, we get the inequality above.

Hence, we have $s^*(\theta) = \theta$ in this case.

Case 2: We show that $\widehat{s}(\theta) < 0$ would imply $s^*(\theta) = 0$.

Suppose $s^*(\theta) > 0$. Then we have $s^*(\theta) > \widehat{\theta}$. By similar argument we know that $m_s^*(\theta) > \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned} \frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\ &< -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\ &= 0, \end{aligned}$$

which implies $s^*(\theta) = 0$. This is a contradiction. Hence, we have $s^*(\theta) = 0$ in this case.

Case 3: We show that $0 \leq \widehat{s}(\theta) \leq \theta$ would imply $s^*(\theta) = \widehat{s}(\theta)$.

Suppose $\widehat{s}(\theta) < s^*(\theta)$. Then similar argument suggests $r(\theta)/m_s^*(\theta) < 0$, which implies $s^*(\theta) = 0 < \widehat{s}(\theta)$. This is a contradiction.

Similarly, suppose $s^*(\theta) < \widehat{s}(\theta)$. Similar argument suggests that $r(\theta)/m_s^*(\theta) > 0$, which implies $s^*(\theta) = \theta > \widehat{s}(\theta)$. This is, again, a contradiction. Hence, we have $s^*(\theta) = \widehat{s}(\theta)$ in this case.

This concludes the proof. \square

PROOF OF LEMMA 7. Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. \square

PROOF OF LEMMA 8. We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a $\widetilde{\theta} \geq 0$, such that $s^*(\theta) = 0$ when $0 \leq \theta \leq \widetilde{\theta}$ and $s^*(\theta) = \widehat{s}(\theta)$ when $\theta > \widetilde{\theta}$.

Note that, $s^*(\theta)$ is strictly increasing when $\theta > \widetilde{\theta}$. Also, since we focus on the equilibrium with information acquisition, there must exist a θ'' such that $s^*(\theta'') > k$; otherwise the optimal security would be rejected without information acquisition. Therefore, there exists a $\theta' > \widetilde{\theta}$ such that $s^*(\theta') = \widehat{s}(\theta') = k$. Recall condition (A.1), we have

$$m_s^*(\theta') = \bar{\pi}_s^*.$$

Moreover, notice that we have $s^*(\theta') \in (0, \theta')$, we have

$$\begin{aligned} 0 = r(\theta') &= -m_s^*(\theta') + \mu^{-1} \cdot m_s^*(\theta') \cdot (1 - m_s^*(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\ &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) + \mathbb{E}[r(\theta)], \end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}[r(\theta)] &= -\bar{\pi}_s^* + \mu^{-1} \left(\mathbb{E} \left[\frac{(\theta - s(\theta)) \cdot g''(\bar{\pi}_s^*)}{g''(m(\theta))} \right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E} \left[\frac{1}{g''(m(\theta))} \right] \right) \\
&= -\bar{\pi}_s^* + \mu^{-1} \left(w^* \cdot \left(1 - \mathbb{E} \left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))} \right] \right) / g''(\bar{\pi}_s^*) + w^* \mathbb{E} \left[\frac{1}{g''(m(\theta))} \right] \right) \\
&= -\bar{\pi}_s^* + \frac{\mu^{-1} w^*}{g''(\bar{\pi}_s^*)} \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot w^* .
\end{aligned}$$

We can express the expectation term $\mathbb{E}[r(\theta)]$ in another way. Note that, for any $\theta \in [0, \tilde{\theta}]$, by definition we have

$$\begin{aligned}
r(\theta) &= -m_s^*(\theta) + \mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&= -\hat{m}_s(\tilde{\theta}) + \mu^{-1} \cdot \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \cdot (\theta - 0 - \theta^* + \theta^* + w^*) \\
&= r(\tilde{\theta}) - \mu^{-1} \cdot \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \cdot (\tilde{\theta} - \theta) \\
&= -\mu^{-1} \cdot \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \cdot (\tilde{\theta} - \theta) .
\end{aligned}$$

Also, as $s^*(\theta) = \hat{s}(\theta)$ for any $\theta > \tilde{\theta}$, we have $r(\theta) = 0$ for all $\theta > \tilde{\theta}$. Hence,

$$\mathbb{E}[r(\theta)] = -\mu^{-1} \cdot \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) .$$

Therefore, we have

$$\mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) = -\mathbb{E}[r(\theta)] \tag{A.22}$$

$$= \mu^{-1} \cdot \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) . \tag{A.23}$$

Now we take the tangent line of $s^*(\theta)$ at $\theta = \tilde{\theta}$. The tangent line intersects $s = k$ at $\tilde{\theta}'$, which is given by

$$\frac{k}{\tilde{\theta}' - \theta'} = \frac{ds^*(\theta)}{d\theta} \Big|_{\theta'} = 1 - \hat{m}_s(\tilde{\theta}) .$$

Hence, we have

$$\tilde{\theta}' = \tilde{\theta} + \frac{k}{1 - \hat{m}_s(\tilde{\theta})} .$$

Also, note that we have shown that for any $\theta \geq \tilde{\theta}$, we have

$$\frac{ds^*(\theta)}{d\theta} = \frac{d\hat{s}(\theta)}{d\theta} = 1 - \hat{m}_s(\theta) = 1 - m_s^*(\theta) .$$

Hence,

$$\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0.$$

Therefore, $s^*(\theta)$ is strictly concave for $\theta \geq \tilde{\theta}$, and consequently, we also have $\tilde{\theta}' < \theta'$.

As a result, from conditions (A.22) and (A.23), we have

$$\begin{aligned} \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\tilde{\theta}' - k) &< \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) \\ &= \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot \left(\tilde{\theta} + \frac{\hat{m}_s(\tilde{\theta})}{1 - \hat{m}_s(\tilde{\theta})} \cdot k \right). \end{aligned}$$

By Jensen's inequality, we get

$$\bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) > \mathbb{E}[m_s^*(\theta) \cdot (1 - m_s^*(\theta))].$$

Therefore, we have

$$\begin{aligned} \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) &> \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot \left(\tilde{\theta} + \frac{\hat{m}_s(\tilde{\theta})}{1 - \hat{m}_s(\tilde{\theta})} \cdot k \right) \\ &> \mathbb{E}[m_s^*(\theta) \cdot (1 - m_s^*(\theta))] \cdot \left(\tilde{\theta} + \frac{\hat{m}_s(\tilde{\theta})}{1 - \hat{m}_s(\tilde{\theta})} \cdot k \right). \end{aligned}$$

Expand the expectation term above and rearrange, we get

$$\begin{aligned} &\hat{m}_s(\tilde{\theta})^2 \cdot k \cdot \text{Prob}[\theta \leq \tilde{\theta}] + \int_{\tilde{\theta}}^{+\infty} m_s^*(\theta) \cdot (1 - m_s^*(\theta)) d\Pi(\theta) \cdot \left(\tilde{\theta} + \frac{\hat{m}_s(\tilde{\theta})}{1 - \hat{m}_s(\tilde{\theta})} \cdot k \right) \\ &< \hat{m}_s(\tilde{\theta}) \cdot (1 - \hat{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (-\theta) d\Pi(\theta) \\ &\leq 0. \end{aligned}$$

Nevertheless, the left hand side of the above inequality should be positive, which is a contradiction. This concludes the proof. \square

PROOF OF PROPOSITION 4. We first consider the case with a positive transfer price $p > 0$. Suppose the corresponding security $s(\theta) = \theta - p$ is optimal in a generalized security design problem without the non-negative constraint. However, this security can be accommodated by neither Proposition 2 or Proposition 3, which two exclusively characterize the optimal security in the generalized security design problem, a contradiction.

By Corollary 1, we know that the security $s(\theta) = \theta$ that represents transfer with a zero price is not optimal. This concludes the proof. \square

PROOF OF COROLLARY 2. First, note that $s^*(\theta)$ is strictly increasing and continuous. Also, note that there exists a θ'' such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists a unique θ' such that $s^*(\theta') = k$, which ensures that $m_s^*(\theta') = \bar{\pi}_s^*$, and

$$\begin{aligned} r(\theta') &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + w^* \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta')] . \end{aligned}$$

Note that $\mathbb{E}[r(\theta')] > 0$ and $\theta' - s^*(\theta') \geq 0$, we have $\theta' < \hat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\hat{\theta} > \theta' = k$. This concludes the proof. \square

PROOF OF PROPOSITION 5. When we have $\mathbb{E}[\theta] \leq k$ and $\mathbb{E}[\exp(\mu^{-1}(t \cdot \theta - k))] > 1$, according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with positive (but less than one) probability. This concludes the proof. \square

LEMMA 9. *A project is initiated with positive probability in the decentralized economy if and only if it is initiated with positive probability in the corresponding centralized economy.*

PROOF OF LEMMA 9. With the objective function (4.10) in the centralized economy, the entrepreneur's optimal screening rule $m_c^*(\theta)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leq 1 ,$$

will skip the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leq 1 ,$$

and will initiate the project with probability $0 < \bar{\pi}_c^* < 1$, $\bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)]$, if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \text{ and } \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1 ,$$

in which $m_c^*(\theta)$ is determined by

$$\theta - k = \mu \cdot (g'(m_c^*(\theta)) - g'(\bar{\pi}_c^*)) .$$

It is straightforward to observe that, the project is initiated with positive probability in the

frictionless centralized economy if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1. \tag{A.24}$$

Clearly, condition (A.24) is just the same as condition (3.1) in Lemma 1 that gives the investment criterion in a corresponding decentralized production economy. This concludes the proof. \square